Numerical simulation of two-fluid flow on multicores accelerator

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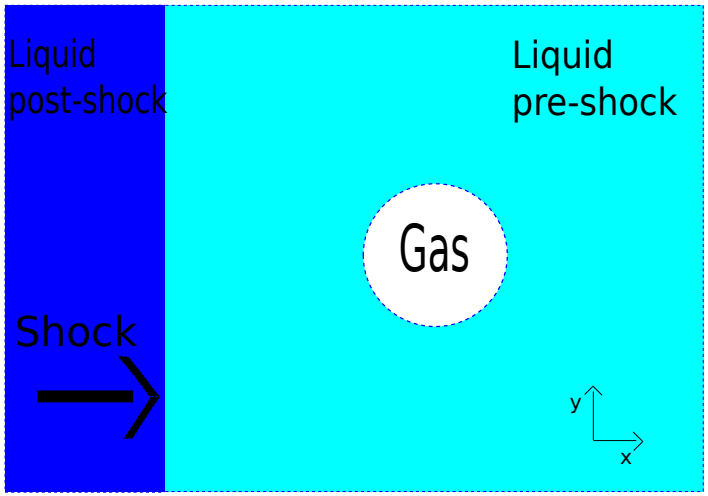
PhD thesis defence, October 28th, 2013

Example of computation

Study case: a shock wave in a liquid interacting with a gas bubble.



Properties of the flow:

- two immiscible fluids,
- the gas and the water are compressible.

Aim: develop a robust and efficient numerical scheme. Adaptation to GPU cluster.

Model

- We consider the 2D compressible Euler equations:

$$\begin{aligned}
\partial_t(\rho) + \partial_x(\rho u) + \partial_y(\rho v) &= 0,\\
\partial_t(\rho u) + \partial_x(\rho u^2 + p) + \partial_y(\rho uv) &= 0,\\
\partial_t(\rho v) + \partial_x(\rho uv) + \partial_y(\rho v^2 + p) &= 0,\\
\partial_t(\rho E) + \partial_x\big((\rho E + p)u\big) + \partial_y\big((\rho E + p)v\big) &= 0,\\
\partial_t(\rho \varphi) + \partial_x(\rho u \varphi) + \partial_y(\rho v \varphi) &= 0,
\end{aligned}$$

  where ρ is the density, (u, v) the velocity vector, E the total energy and φ the gas mass fraction.
- The liquid-gas interface is located at the discontinuities of the gas fraction φ. In the liquid, we have $\varphi = 0$ and in the gas $\varphi = 1$.
- We have 5 equations for 6 unknowns.

Equation of states

- Galilean invariance of the Euler equations:

$$p = p(\tau, e, \varphi),$$

where

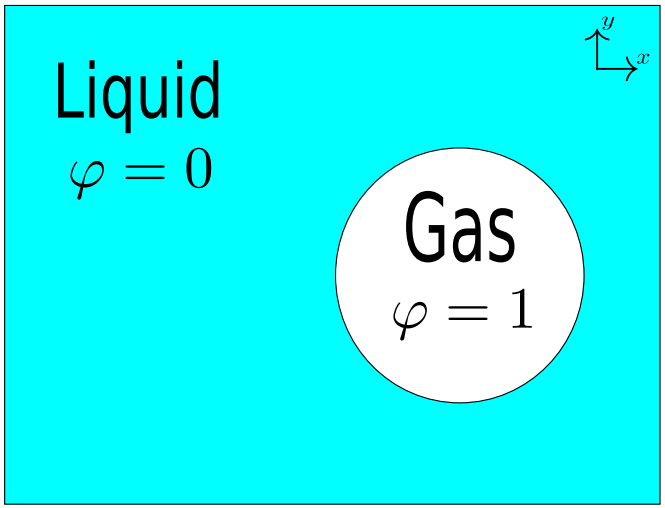
$$\tau = \frac{1}{\rho},$$
$$e = E - \frac{u^2 + v^2}{2}.$$

- The pressure p can be given by a stiffened gas law whose coefficients depend on the gas fraction φ

$$p(\tau, e, \varphi) = \big(\gamma(\varphi) - 1\big)\frac{e}{\tau} - \gamma(\varphi)p_\infty(\varphi).$$

Mixture

- At initial time,



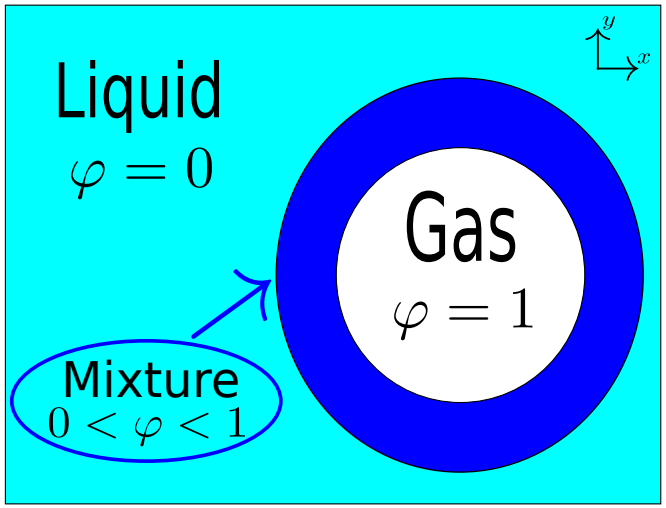
- Theoretically,

$$\partial_t \varphi + u\partial_x \varphi + v\partial_y \varphi = 0.$$

$\rightarrow \varphi = 0$ or $\varphi = 1$ at any time.

- Numerically, we have diffusion

$$0 < \varphi < 1.$$



- In the mixture, [SA99]

$$\frac{1}{\gamma(\varphi) - 1} = \varphi \frac{1}{\gamma_{gas} - 1} + (1 - \varphi) \frac{1}{\gamma_{liq} - 1},$$

$$\frac{\gamma(\varphi) p_\infty(\varphi)}{\gamma(\varphi) - 1} = \varphi \frac{\gamma_{gas} p_{\infty,gas}}{\gamma_{gas} - 1} + (1 - \varphi) \frac{\gamma_{liq} p_{\infty,liq}}{\gamma_{liq} - 1}.$$

Conservative form

The system admits a conservative form:

$$\partial_t W + \partial_x F(W) + \partial_y G(W) = 0,$$

where

$$\begin{aligned} W &= (\rho, \rho u, \rho v, \rho E, \rho \varphi)^T, \\ F(W) &= \left(\rho u, \rho u^2 + p, \rho uv, (\rho E + p)u, \rho \varphi u\right)^T, \\ G(W) &= \left(\rho v, \rho uv, \rho v^2 + p, (\rho E + p)v, \rho \varphi v\right)^T. \end{aligned}$$

Hyperbolicity of the system

The system is hyperbolic in each direction [GR91]. For each $\mathbf{n} = (n_1, n_2)^T$, the matrix $n_1 D_W F(W) + n_2 D_W G(W)$ is **diagonalizable** with real eigenvalues on the domain

$$\Omega := \left\{ W \in \mathbb{R}^5,\ \rho > 0,\ \varphi \in [0;1],\ p\left(\tau, e, \varphi\right) + p_\infty(\varphi) > 0 \right\},$$

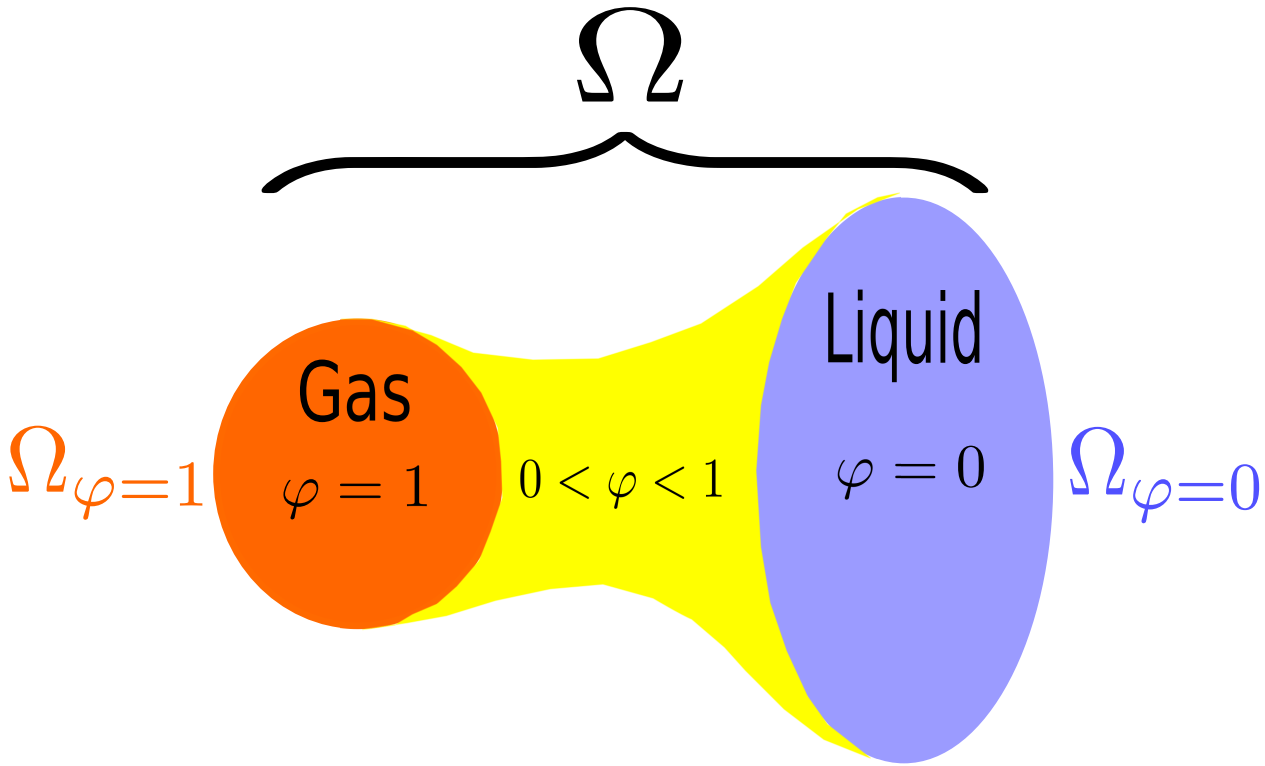
and admits the **eigenvalues**:

$$\lambda_1 = (u,v)^T.\mathbf{n} - c, \qquad \lambda_2 = \lambda_3 = \lambda_4 = (u,v)^T.\mathbf{n}, \qquad \lambda_5 = (u,v)^T.\mathbf{n} + c,$$

where for $W \in \Omega$, the **sound speed** c is given by

$$c(\tau, e, \varphi) = \sqrt{\gamma(\varphi)\tau\big(p\left(\tau, e, \varphi\right) + p_\infty(\varphi)\big)}.$$

Non convexity of the hyperbolic set Ω



- Ω is not a convex set.
- $\Omega_{\varphi=0}$ and $\Omega_{\varphi=1}$, defined by

  $$\Omega_{\varphi=\varphi_0} := \left\{ W \in \mathbb{R}^5,\ \rho > 0,\ \varphi = \varphi_0,\ p\left(\tau, e, \varphi_0\right) + p_\infty(\varphi_0) > 0 \right\}.$$

  are convex sets.

Aim

- For approximating the 2D system, we consider directional splitting (Godunov [G59], Strang [S68]).
  $\rightarrow$ We solve successively the 1D systems

$$\partial_t W + \partial_x F(W) = 0, \quad \partial_t W + \partial_y G(W) = 0.$$

- From rotational invariance we only have to construct an approximation of

$$\partial_t W + \partial_x F(W) = 0,\ x \in [a;b],\ t > 0,$$
$$W(x,0) = W_0(x),$$

with some boundary conditions at a and b.

Riemann problem

- Riemann problem

$$\partial_t W + \partial_x F(W) = 0,$$

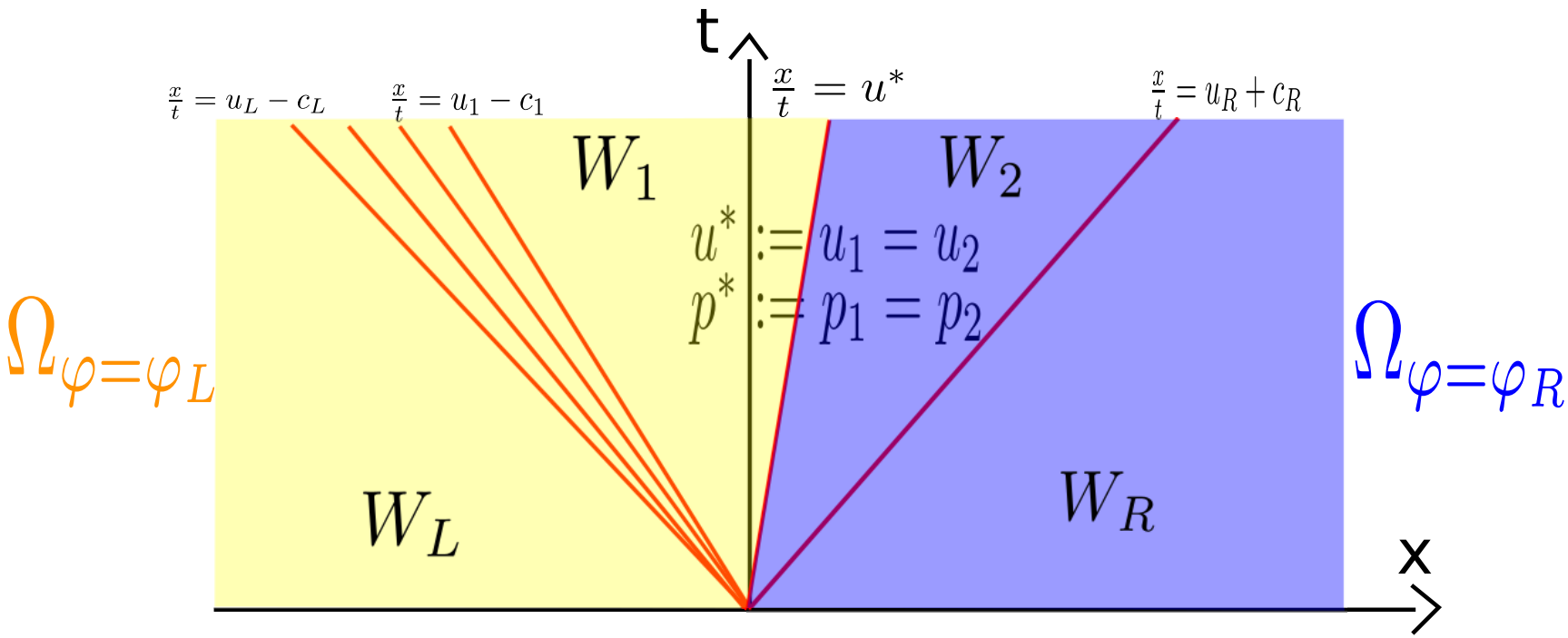
$$W(x,0) = \begin{cases} W_L, & \text{if } x < 0, \\ W_R, & \text{otherwise.} \end{cases}$$

- Even if Ω is not convex, there exists only one, self-similar, global, entropy solution [GR91]

$$W(x,t) = R(W_L, W_R, x/t).$$

- It is entirely defined by 4 constant states W_L, $W_1 \in \Omega_{\varphi=\varphi_L}$, $W_2 \in \Omega_{\varphi=\varphi_R}$ and W_R separated by self-similar waves: rarefaction, shock or contact.

Structure of the solution to the Riemann problem



Example of solution to the Riemann problem $R(W_L, W_R, x/t)$:
a 1-rarefaction, a 2,3,4-contact and a 5-shock.
The exact solution lies in the non convex set $\Omega_{\varphi=\varphi_L} \cup \Omega_{\varphi=\varphi_R}$.

Godunov scheme

- We consider a space step $\Delta x = \frac{b-a}{N}$ and points

$$x_{i-\frac{1}{2}} = a + i \times \Delta x.$$

- The cell (or finite volume) C_i is the interval $]x_{i-\frac{1}{2}}; x_{i+\frac{1}{2}}[$.
- We note x_i the middle of the cell C_i

$$x_i = \frac{x_{i-\frac{1}{2}} + x_{i+\frac{1}{2}}}{2}.$$

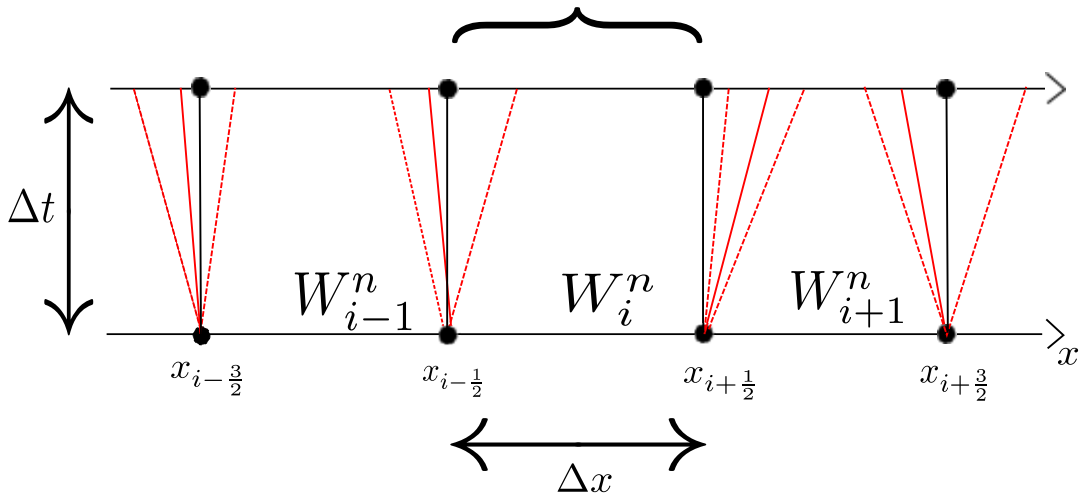
- We start with a piecewise constant approximation of $W(x,0)$ made of cell averages

$$W_i^0 = \frac{1}{\Delta x} \int_{C_i} W(x,0) dx.$$

Time evolution of the Godunov scheme

$$W_i^{n+1} = \frac{1}{\Delta x} \int_{C_i} V(x, \Delta t) dx$$

Averaging step



First step :
We compute the exact solution of

$$\begin{cases} \partial_t V + \partial_x F(V) = 0, \\ V(x,0) = W_i^n, \ x_{i-\frac{1}{2}} < x < x_{i-\frac{1}{2}} \end{cases}$$

$\rightarrow$ if $x_i < x < x_{i+1}$,

$$V(x, \Delta t) = R(W_i^n, W_{i+1}, \frac{\Delta x}{\Delta t}) \in \Omega.$$

Stability issue: Ω is not convex

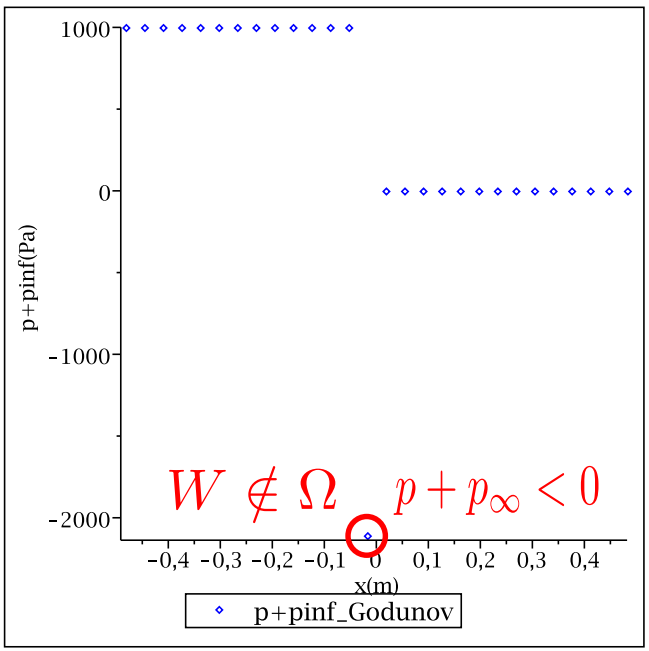
- The exact solution $V(x, \Delta t) \in \Omega$ for all $x \in [a; b]$.
- As Ω is not convex, the averaging

$$W_i^{n+1} = \frac{1}{\Delta x} \int_{C_i} V(x, \Delta t) dx,$$

may produce values outside Ω.

- For example:

Quantities	$x < 0$	$x > 0$
$\rho(kg.m^{-3})$	10	1
$u(m.s^{-1})$	-50	0
$p(Pa)$	1	1
φ	1	0
γ	4.4	3
$p_\infty(Pa)$	1000	0



see also [MHB10].

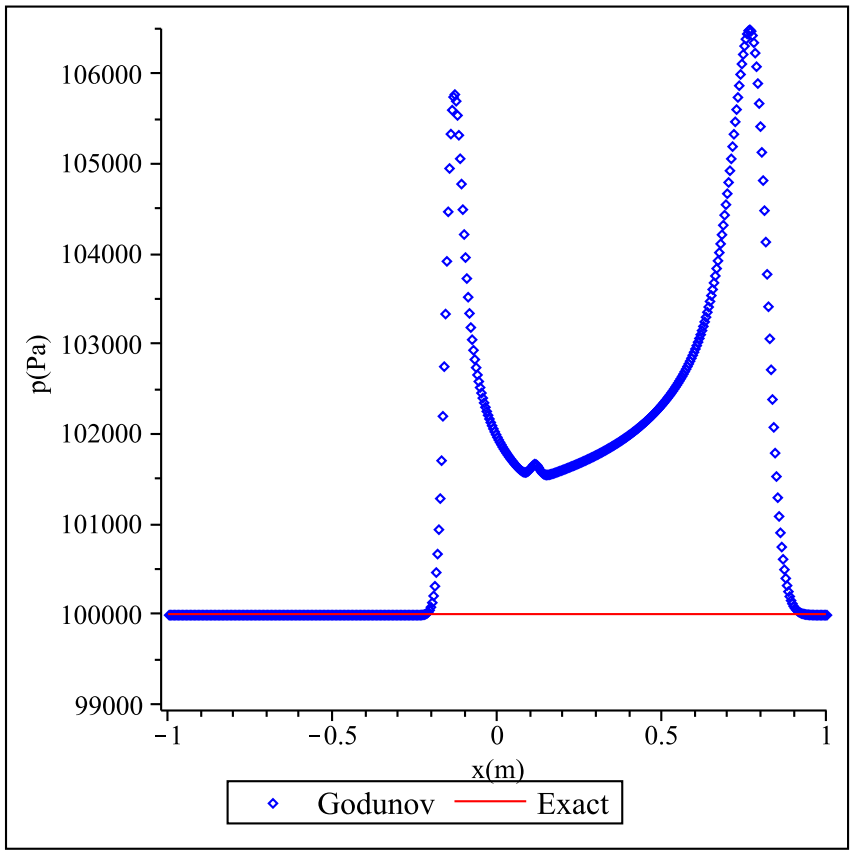
Spurious pressure oscillations at the two-fluid interface

Regardless of the stability problem, we observe velocity-pressure spurious oscillations at the two-fluid interface [A88].

For example, with

Quantities	$x < 0$	$x > 0$
$\rho(kg.m^{-3})$	10	1
$u(m.s^{-1})$	50	50
$p(Pa)$	1e5	1e5
φ	1	0
γ	1.4	1.1
$p_\infty(Pa)$	0	0

we obtain:



Aim of the thesis

- Solve the stability problems due to the non convexity of Ω for a stiffened gas pressure law.
- Solve the problem of the pressure oscillations at the two-fluid interface.
- Extend the method to study the oscillations of a spherical bubble of gas in a liquid phase (well-balanced solver).
  $\rightarrow$ Not presented here.
- Develop a robust and efficient 2D numerical scheme.
- Implement the numerical scheme on Graphics Processing Units (GPU) and on GPU clusters.

How to remove the spurious pressure oscillations?

Previous works on the spurious pressure oscillations issue:

- use a Eulerian approach with a **non conservative flux** at the two-fluid interface (Abgrall [A88], Karni [Ka94], Fedkiw [F99], Saurel-Abgrall [SA99], Abgrall-Karni [AK01], *etc*),
- use a Lagrangian-projection scheme and project the pressure at interface (Barberon-Helluy-Rouy [BHR07], Chalons-Coquel [CC10]). The projection is **non conservative**.

$\rightarrow$ We obtain non conservative schemes.

How to remove the stability problems?

Previous works on the lack of stability caused by the non-convexity of the hyperbolic set:

- Other pressure law (Callen [C85], Croisille [C91], Chanteperdrix, Villedieu and Vila [CVV02], *etc*).
  $\rightarrow$ The proof of convexity is not provided.
- Chalons and Goatin [CG08] proposed a numerical method with a random sampling for computing phase transitions in traffic flow modeling.

How to solve these two issues?

- First approach: construct another pressure law $\tilde{p}$ to obtain a convex domain of hyperbolicity $\tilde{\Omega}$.
  $\rightarrow$ Stable and conservative numerical scheme **but** the spurious pressure oscillations are not removed.

- Second approach: keep a stiffened gas pressure law for the liquid and the gas and use a numerical strategy in order to avoid diffusion on φ and to preserve the non convex domain Ω.
  $\rightarrow$ Stable and conservative numerical scheme **without** spurious pressure oscillations.

First approach: construct an other pressure law $\tilde{p}(\tau, e, \varphi)$

Procedure

- The gas (resp. the liquid) is described by the **massic entropy** $s_1 = s_1(\tau_1, e_1)$ (resp. $s_2 = s_2(\tau_2, e_2)$) linked to the pressure p_i of phase i

$$p_i(\tau_i, e_i) = \frac{\partial_{\tau_i} s_i}{\partial_{e_i} s_i}.$$

- The mixture pressure $\tilde{p}(\tau, e, \varphi)$ is derived from the mixture entropy $s = s(\tau, e, \varphi)$ by the relation

$$\tilde{p}(\tau, e, \varphi) = \frac{\partial_\tau s}{\partial_e s}.$$

$\rightarrow$ We need a procedure for constructing the mixture entropy s from s_1 and s_2.

Mixture entropy and pressure law

- At equilibrium, the immiscible mixture entropy $s(\tau, e, \varphi)$ is given by

$$s(\tau, e, \varphi) = \sup_{0 \leq \alpha, z \leq 1} \left(\varphi s_1\left(\frac{\alpha}{\varphi}\tau, \frac{z}{\varphi}e\right) + (1-\varphi) s_2\left(\frac{1-\alpha}{1-\varphi}\tau, \frac{1-z}{1-\varphi}e\right)\right).$$

- From $\tilde{p} = \frac{\partial_\tau s}{\partial_e s}$, we deduce the pressure law

$$\tilde{p}(\tau, e, \varphi) = \big(\gamma(\tau, e, \varphi) - 1\big)\frac{e}{\tau} - \gamma(\tau, e, \varphi) p_\infty(\tau, e, \varphi).$$

- In the liquid ($p_{\infty,liq} > 0$, $\varphi = 0$), the pressure law is modified, even if there is no gas. Indeed

$$\tilde{p}(\tau, e, 0) = \begin{cases} 0 & , \text{ if } e \leq \frac{\gamma_{liq} p_{\infty,liq}}{\gamma_{liq}-1}\tau, \\ (\gamma_{liq} - 1)\frac{e}{\tau} - \gamma_{liq} p_{\infty,liq}, & \text{otherwise.} \end{cases}$$
$$\neq p_{liq}(\tau, e).$$

$\rightarrow$ Negative pressure are replaced by 0.

Properties of the system with the mixture pressure $\tilde{p}$

- With the mixture entropy s, we construct a Lax entropy

$$\tilde{\Omega} \to \mathbb{R},$$

$$W = (\rho, \rho u, \rho v, \rho E, \rho\varphi) \mapsto -\rho s\Big(\frac{1}{\rho}, \frac{2\rho E - (\rho u)^2 + (\rho v)^2}{2\rho}, \varphi\Big),$$

for the system $\partial_t W + \partial_x F(W) = 0$ coupled with $\tilde{p}$.
Here, $\tilde{\Omega}$ is a convex set.

- From Mock theorem, we deduce the hyperbolicity of the system

$$\partial_t W + \partial_x F(W) = 0,$$

coupled with $\tilde{p}(\tau, e, \varphi)$ on the convex set $\tilde{\Omega}$.

Advantages and drawbacks of $\tilde{p}$

Advantage:

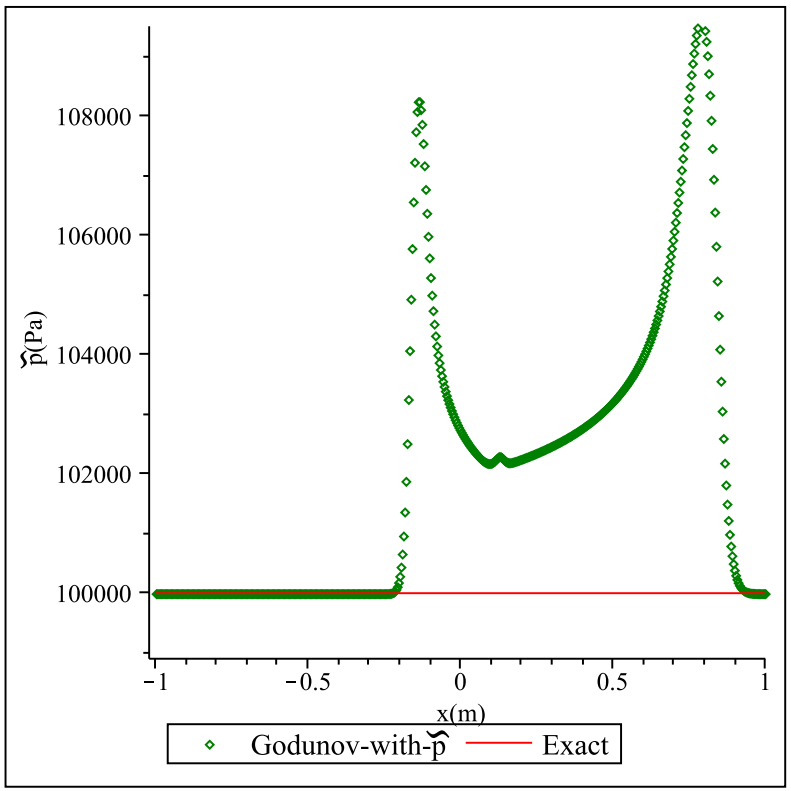
- $\tilde{\Omega}$ is convex
  $\rightarrow$ the Godunov scheme is conservative and stable.

Drawback:

- For some parameters, the liquid degenerates to a pressureless gas.
  $\rightarrow$ Theoretical and numerical difficulties.
- The problem of the spurious oscillations at the two-fluid interface is not solved.

$\rightarrow$ We decide to concentrate on the second idea.

$\longrightarrow$



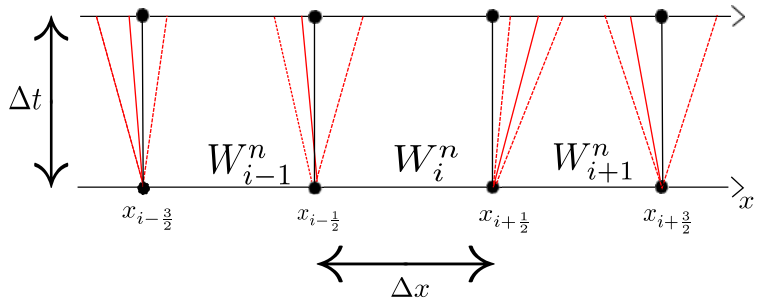
Second approach: construct a numerical method to preserve the non convex domain Ω

Problem of the Godunov scheme

- Now, the gas and the liquid satisfy stiffened gas pressure law again.
- Godunov scheme:

$$W_i^{n+1} = \frac{1}{\Delta x} \int_{C_i} V(x, \Delta t) dx$$

Averaging step



First step :
We compute the exact solution of

$$\begin{cases} \partial_t V + \partial_x F(V) = 0, \\ V(x,0) = W_i^n, \ x_{i-\frac{1}{2}} < x < x_{i-\frac{1}{2}} \end{cases}$$

$\rightarrow$ if $x_i < x < x_{i+1}$,

$$V(x, \Delta t) = R(W_i^n, W_{i+1}, \frac{\Delta x}{\Delta t}) \in \Omega.$$

$\rightarrow V(x, \Delta t)$ lies in the non convex set Ω.
$\rightarrow$ spurious pressure oscillations and lack of stability arise from the averaging step.

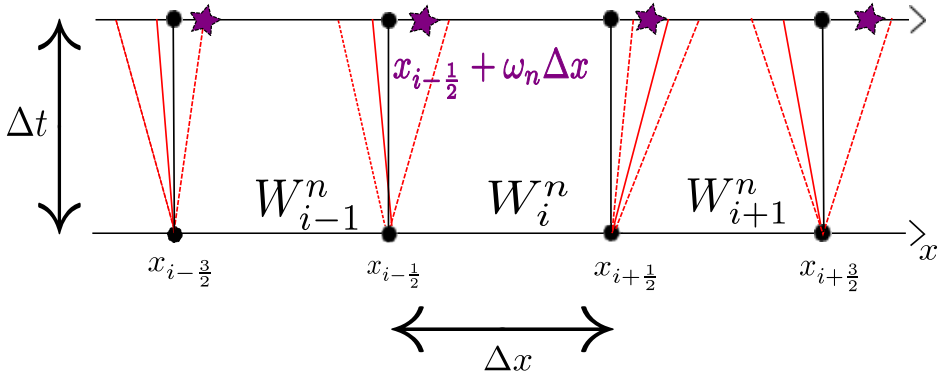
- Conclusion: replace the averaging step.

Glimm scheme [G65]

We take a pseudo-random number $\omega_n \in [0;1[$ and we replace the averaging step of the Godunov scheme.

$$W_i^{n+1} = V(x_{i-\frac{1}{2}} + \omega_n \Delta x, \Delta t)$$

Averaging step



First step :
We compute the exact solution of

$$\begin{cases} \partial_t V + \partial_x F(V) = 0, \\ V(x,0) = W_i^n, \ x_{i-\frac{1}{2}} < x < x_{i-\frac{1}{2}} \end{cases}$$

$\rightarrow$ if $x_i < x < x_{i+1}$,

$$V(x, \Delta t) = R(W_i^n, W_{i+1}, \frac{\Delta x}{\Delta t}) \in \Omega.$$

Advantages and drawbacks of the Glimm scheme

Advantages:

- It does not introduce numerical mixture.
- No spurious pressure oscillations at the two-fluid interface.
- It is **stable**, it preserves the hyperbolic set without diffusion

$$\Omega_{\varphi=0} \cup \Omega_{\varphi=1}.$$

Drawbacks:

- Exact Riemann solver needed.
- Noisy rarefaction waves.
- No extension to higher dimensions (Colella [C78]).

→ We do not follow this approach and introduce the class of the ALE-projection scheme.

ALE-projection scheme

The Arbitrary Lagrangian Eulerian (ALE)-projection scheme includes two steps:

- the ALE-step: we solve the problem on a moving mesh between time t^n and $t^{n+1,-}$.
  - The boundary $x_{i+\frac{1}{2}}$ moves at an arbitrary velocity $\xi^n_{i+\frac{1}{2}}$ between t^n and $t^{n+1,-}$

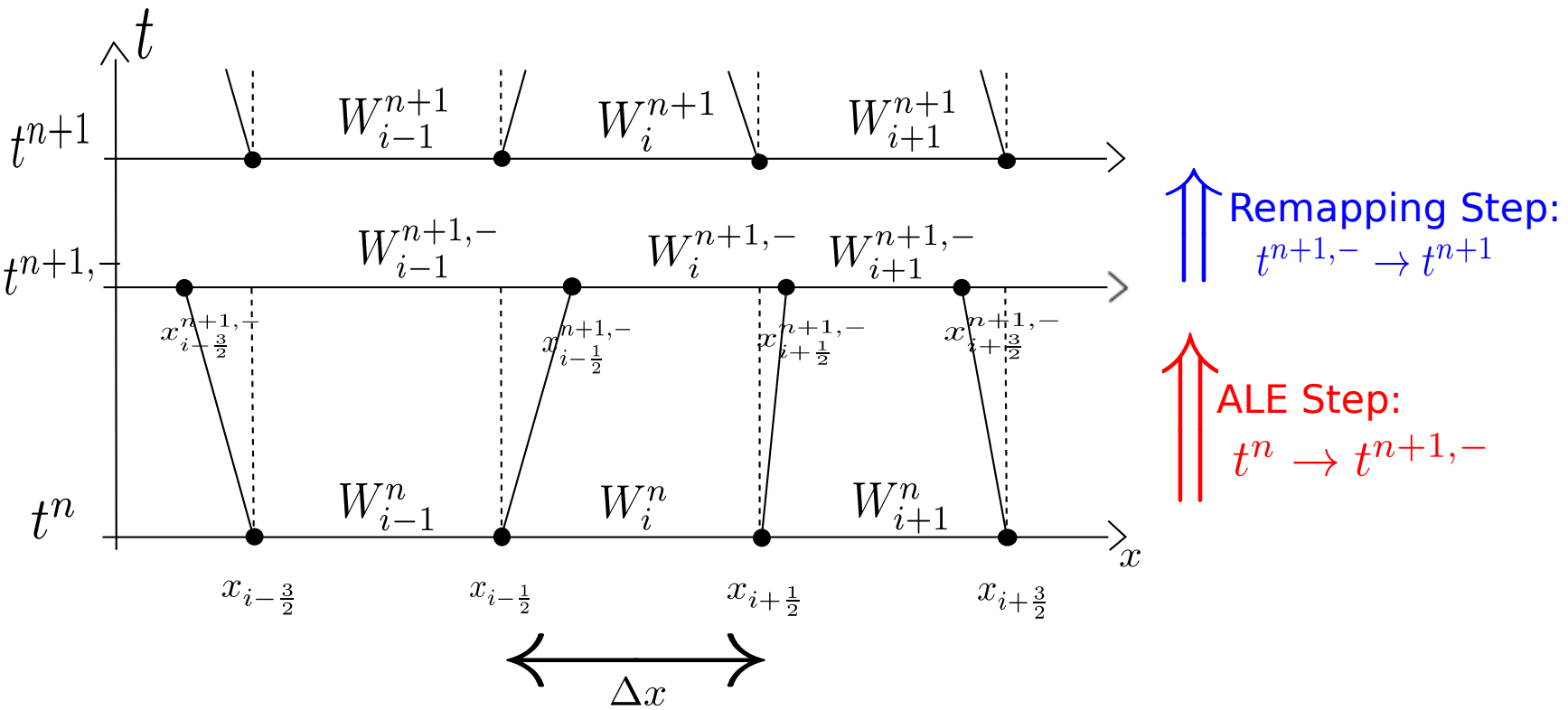
$$x^{n+1,-}_{i+\frac{1}{2}} = x_{i+\frac{1}{2}} + \Delta t\, \xi^n_{i+\frac{1}{2}}.$$

  - We obtain a constant piecewise approximation $W^{n+1,-}$ of $W(\cdot, t_{n+1})$ on cells

$$C^{n+1,-}_i = \left]x^{n+1,-}_{i+\frac{1}{2}}; x^{n+1,-}_{i+\frac{1}{2}}\right[,$$

- the projection step: we project $W^{n+1,-}$ on the original cell C_i.

Structure of ALE-projection scheme



ALE step: expression of $W_i^{n+1,-}$

$W_i^{n+1,-}$ is given by the finite volume scheme

$$\Delta x_i^{n+1,-} W_i^{n+1,-} = \Delta x W_i^n - \Delta t \Big(F(W_i^n, W_{i+1}^n, \xi_{i+\frac{1}{2}}^n) - F(W_{i-1}^n, W_i^n, \xi_{i-\frac{1}{2}}^n) \Big)$$

where Δt satisfies some CFL condition and $F(W_L, W_R, \xi)$ is the numerical flux, given by

$$F(W_L, W_R, \xi) = F\big(R(W_L, W_R, \xi)\big) - \xi R(W_L, W_R, \xi).$$

We have to choose:

- the boundary velocity $\xi_{i+\frac{1}{2}}^n$,
- the projection to go back to the original grid.

Lagrange + Glimm remap

- We choose $\xi^n_{i+\frac{1}{2}} = u^*$ where u^* is the contact discontinuity velocity in the resolution of the Riemann problem $R(W^n_i, W^n_{i+1}, \cdot)$.
- We perform a random sampling

$$W^{n+1}_i = W^{n+1,-}(x_{i-\frac{1}{2}} + \omega_n \Delta x, \Delta t),$$

  where $\omega_n \in [0; 1[$ is a pseudo-random number.
- Properties of the scheme:
  - constant velocity-pressure states are preserved,
  - stability of the non convex hyperbolic set $\Omega_{\varphi=0} \cup \Omega_{\varphi=1}$,

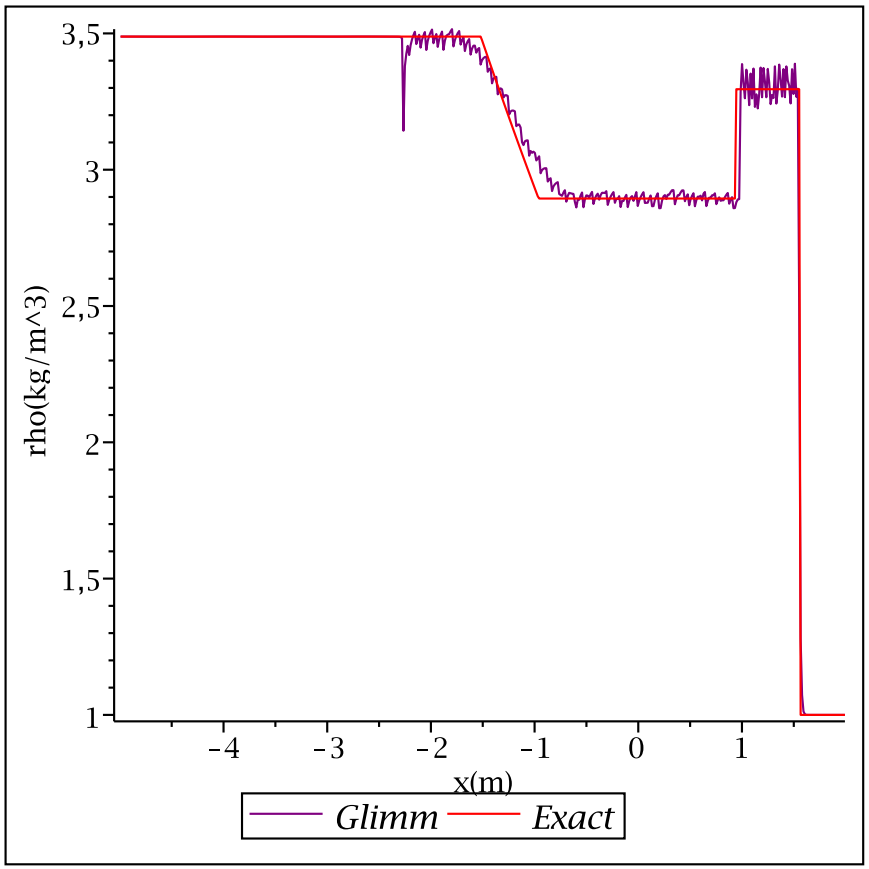
BUT...

Lagrange + Glimm remap: BV instabilities

We consider the following shock-interface interaction

Quantities	$x < -4$	$-4 < x < 1$	$x > 1$
$\rho(kg.m^{-3})$	3.488	2	1
$u(m.s^{-1})$	1.13	-1	-1
$p(Pa)$	23.33	2	2
φ	1	1	0
γ	1.4	1.4	2
$p_\infty(Pa)$	0	0	7

We observe BV instabilities due to the strong shock:



ALE+Glimm projection

- We choose to move the boundary $x_{i+\frac{1}{2}}$ at the speed of the fluid only at the two-fluid interface

$$\xi^n_{i+\frac{1}{2}} = \begin{cases} u^*_{i+\frac{1}{2}} & \text{if} \quad \left(\varphi^n_i - \frac{1}{2}\right)\left(\varphi^n_{i+1} - \frac{1}{2}\right) < 0, \\ 0 & \text{otherwise.} \end{cases}$$

- We perform a random sampling

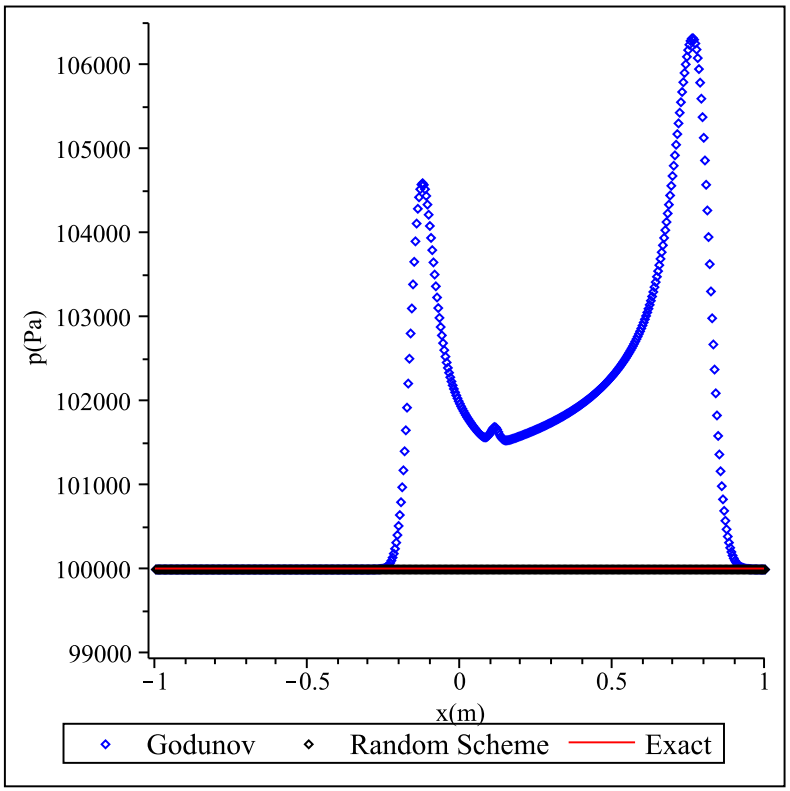
$$W^{n+1}_i = W^{n+1,-}(x_{i-\frac{1}{2}} + \omega_n \Delta x, \Delta t),$$

where $\omega_n \in [0; 1[$ is a pseudo-random number.

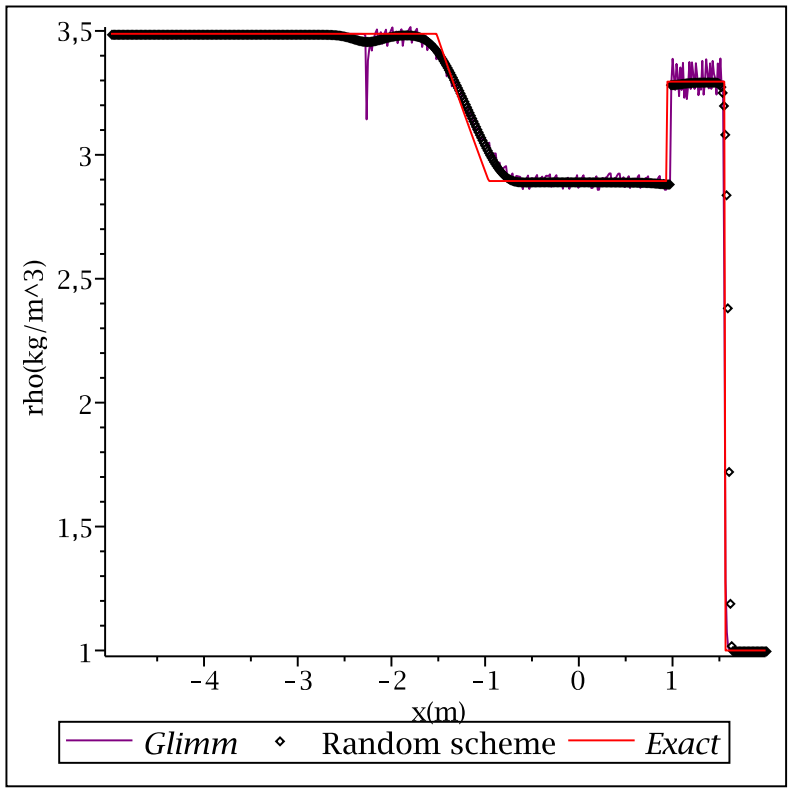
$\rightarrow$ We denote this scheme the "Random Scheme" (RS).

Results with the Random Scheme (RS)

No spurious pressure oscillations at the two-fluid interface:



No BV instabilities:



Properties of the Random Scheme

The Random Scheme has the following properties:

- it preserves hyperbolic set without diffusion $\Omega_{\varphi=0} \cup \Omega_{\varphi=1}$,
- it does not diffuse the gas mass fraction φ,
- it is statistically conservative,
- it satisfies statistically a discrete entropy inequality,
- it does not introduce spurious oscillations at the two-fluid interface,
- it handles vacuum.

A 1D test: comparison with other schemes

We compare the Random Scheme with other schemes:

- the "Ghost Fluid for the poor" (GF) algorithm of Abgrall-Karni [AK01],
- the "SA" algorithm of Saurel-Abgrall [SA99],
- the "NonConsPhi" scheme that includes two steps:
  - a Lagrangian step: as the Lagrange + Glimm scheme,
  - a projection step: we use a conservative projection for ρ, ρu, ρv and ρE and for φ, we take:

$$\varphi_i^{n+1} = \varphi_i^{n+1,-} - \frac{\Delta t}{\Delta x} \Big(\max(u_{i-\frac{1}{2}}, 0)(\varphi_i^{n+1,-} - \varphi_{i-1}^{n+1,-}) + \min(u_{i+\frac{1}{2}}, 0)(\varphi_{i+1}^{n+1,-} - \varphi_i^{n+1,-}) \Big).$$

- the "CC" algorithm of Chalons-Coquel [CC10]. Lagrange-projection scheme with a projection of the pressure and a random sampling on φ at the two-fluid interface.

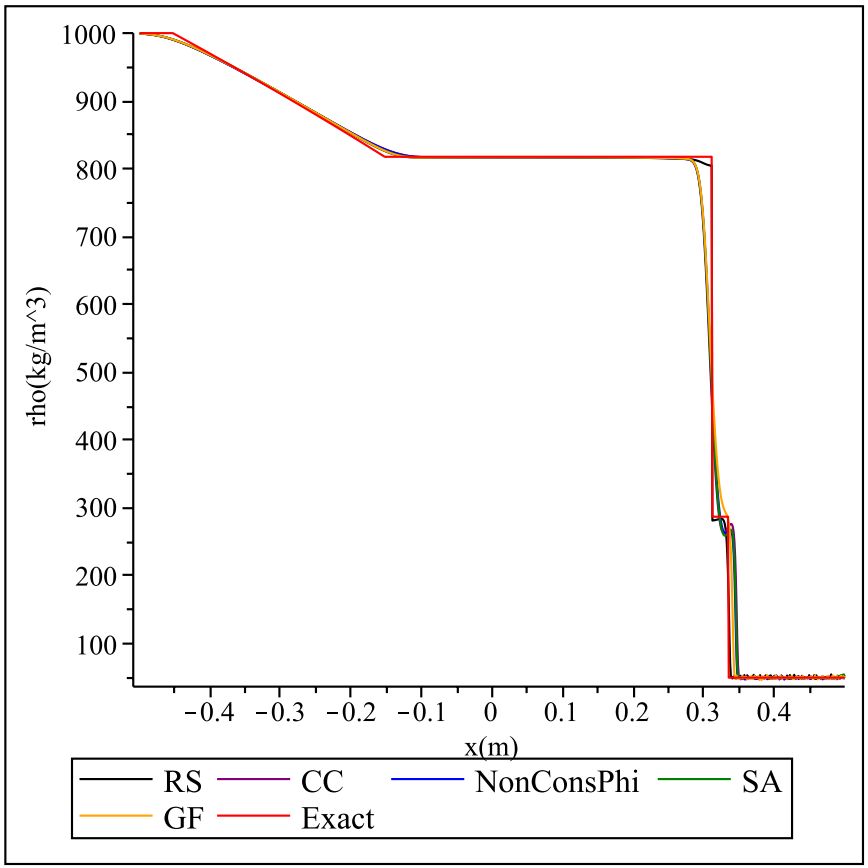
A 1D test

We test the Random Scheme on a water-air shock tube with discontinuous normal velocity.

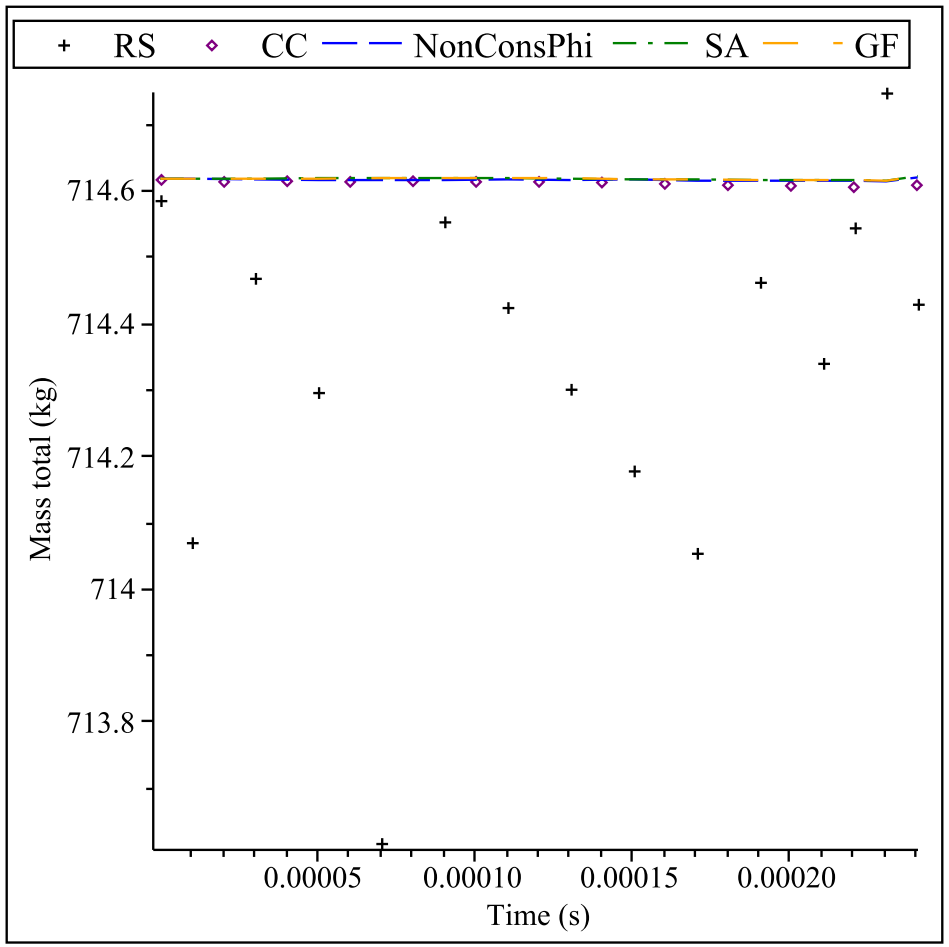
Initial condition:

Quantities	x<0.2	x>0.2
$\rho(kg.m^{-3})$	1000	50
$u(m.s^{-1})$	0	0
$v(m.s^{-1})$	1000	-5000
$p(Pa)$	1e9	1e5
φ	0	1
γ	4.4	1.4
$p_\infty(Pa)$	6.8e8	0

We plot the numerical and the exact solution at time $t = 240\mu m$ with 1000 cells:

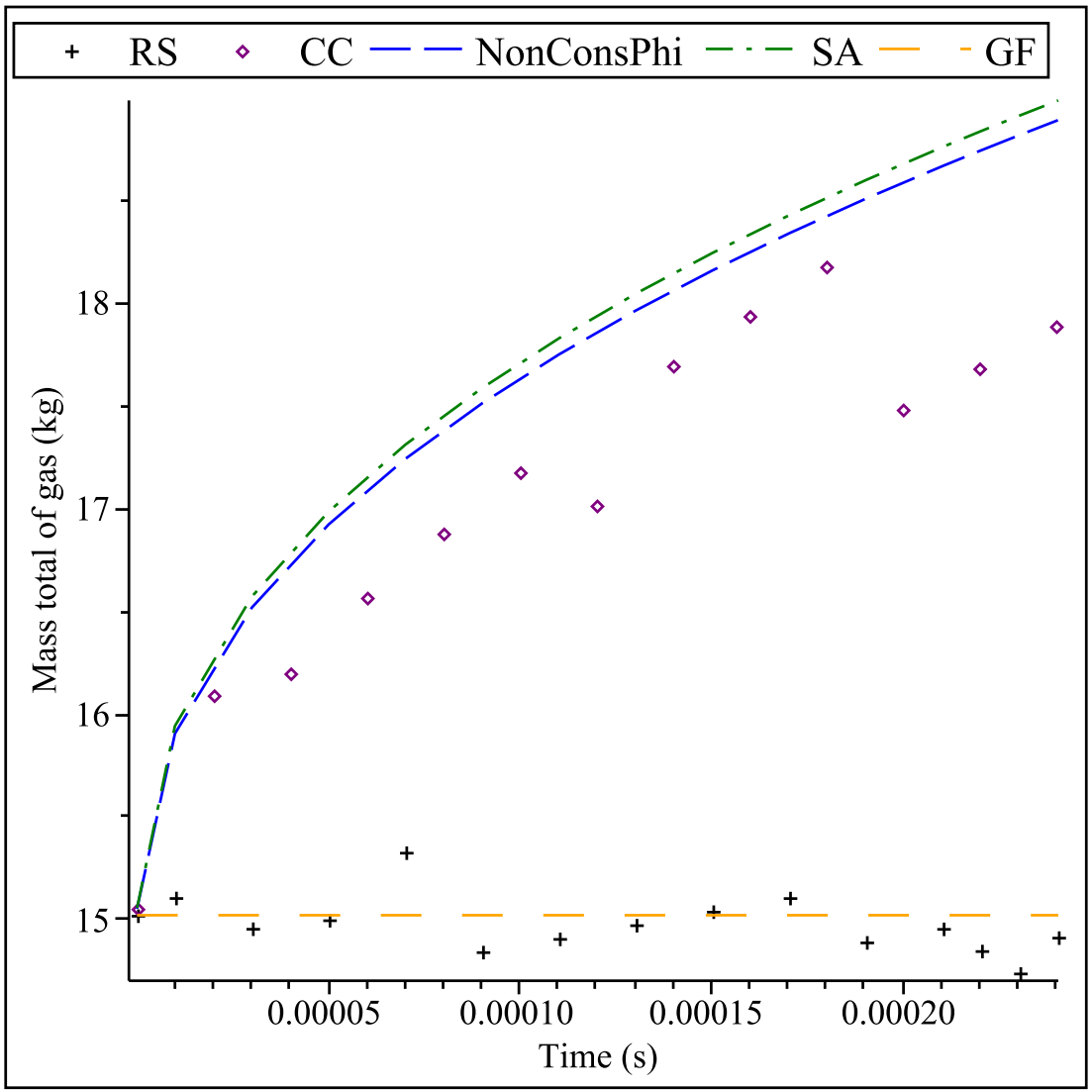


Conservation of the total mass (gas+liquid) until $240\mu s$

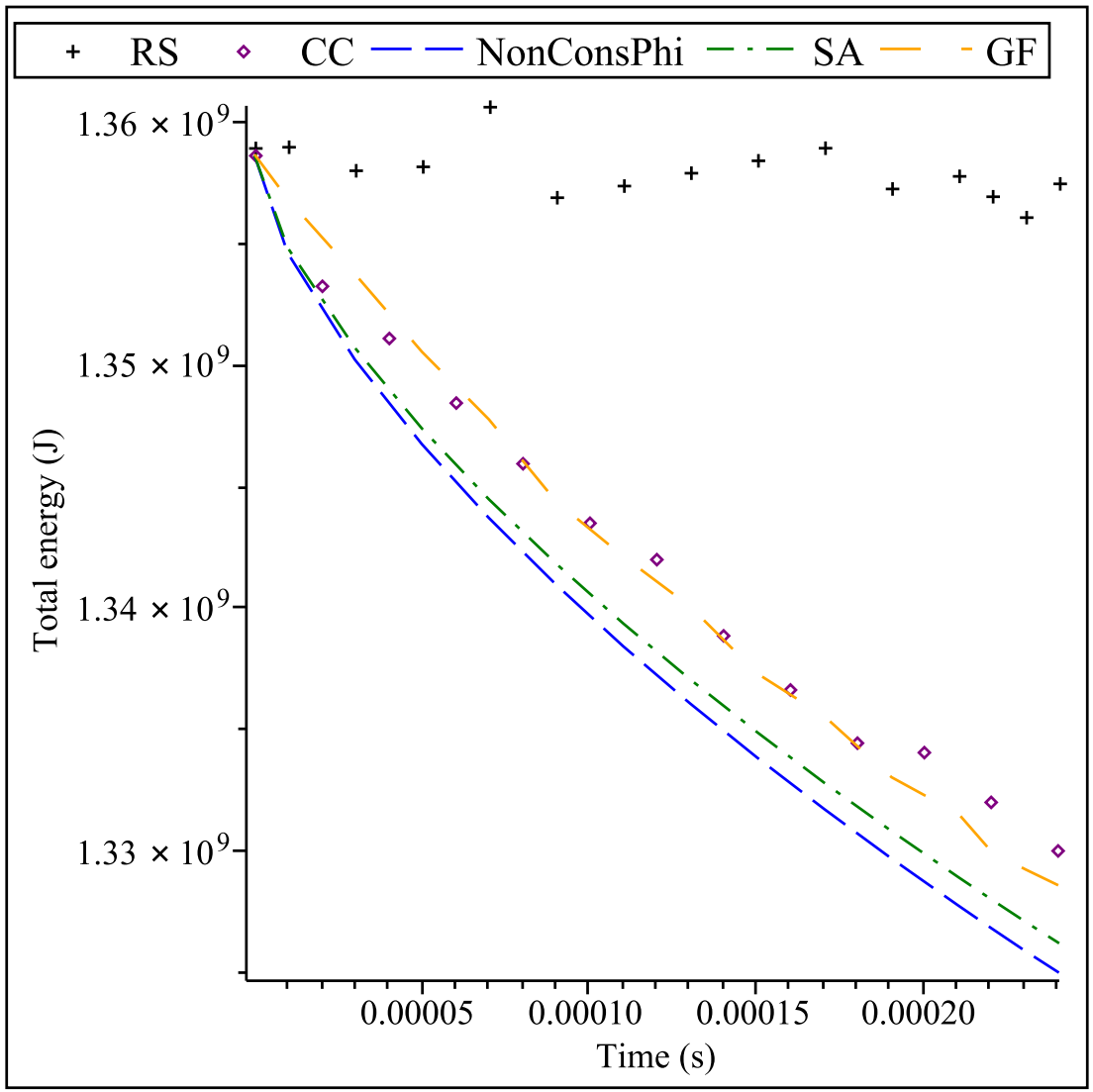


$\rightarrow$ Error less than 0.1%.

Conservation of the mass of gas until $240\mu s$



Conservation of the total energy until $240\mu s$



Extension to 2D with directional splitting

- For solving

$$\begin{cases} \partial_t W + \partial_x F(W) + \partial_y G(W) = 0, \\ W(x, y, t = 0) = W_0(x, y), \end{cases}$$

  between time $t = 0$ and $t = \Delta t$, we use directional splitting (Godunov [G59] or Strang [S68]).

- Colella [C78] showed that the directional splitting coupled with the Glimm scheme does not converge (because of the nonlinear waves).
  $\rightarrow$ Does the directional splitting work with the Random Scheme ?

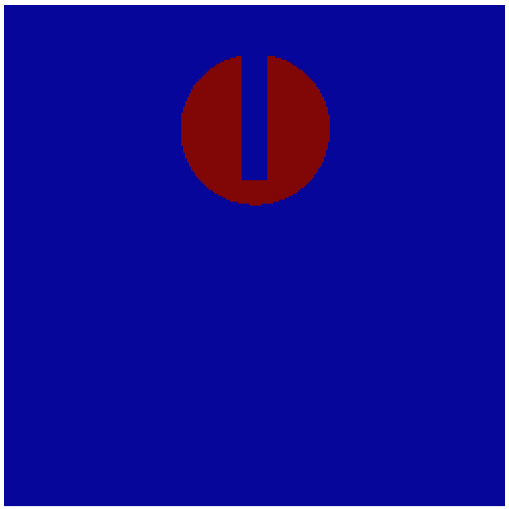
Test of Zalesak [Z79]

We consider a solid body rotation. We choose

$$u = -\Omega(y - y_0), \qquad v = \Omega(x - x_0),$$

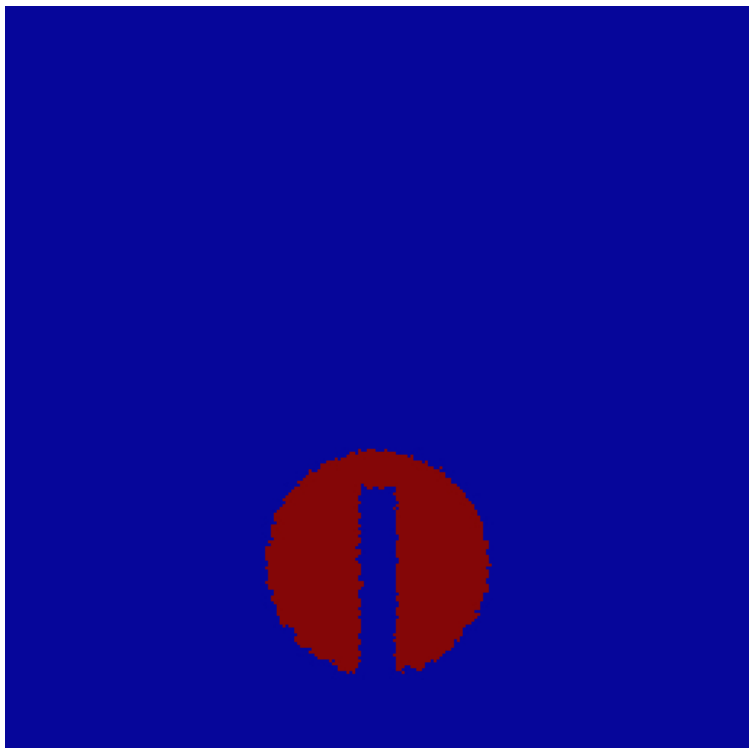
where $\Omega = \frac{2\pi}{628}$ is the angular velocity and (x_0, y_0) is the axis of rotation.

- Initial time:

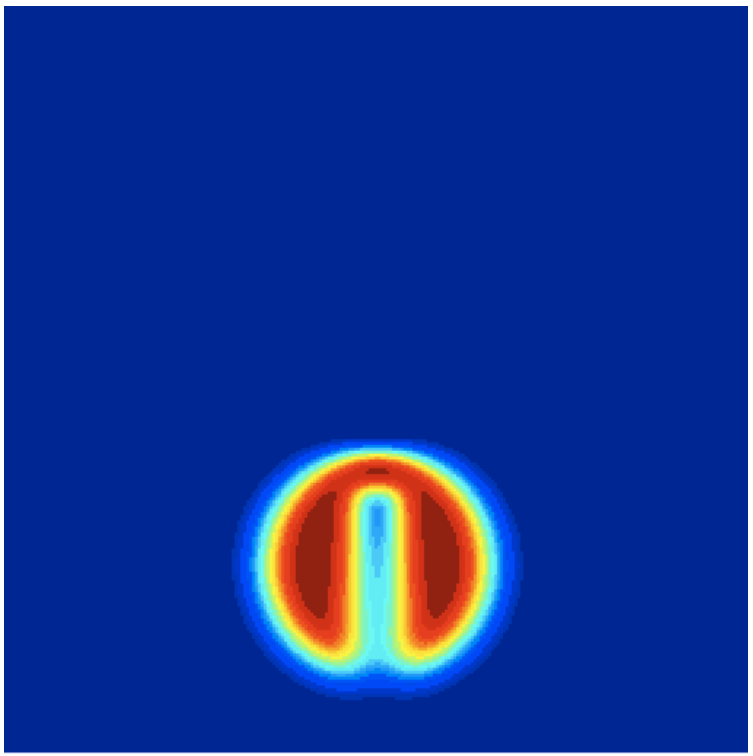


Test of Zalesak [Z79]: $\frac{1}{2}$ revolution

Random Scheme:

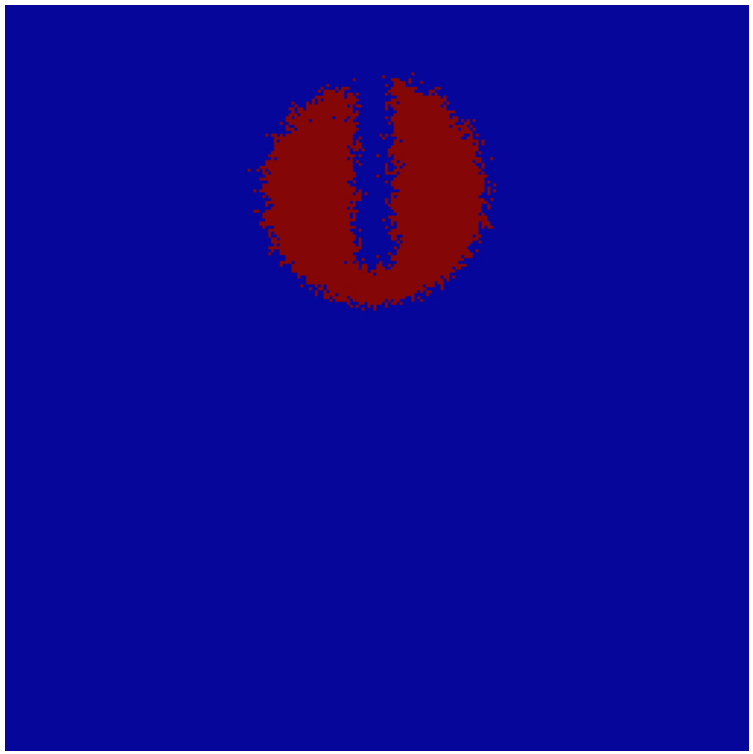


Conservative scheme:

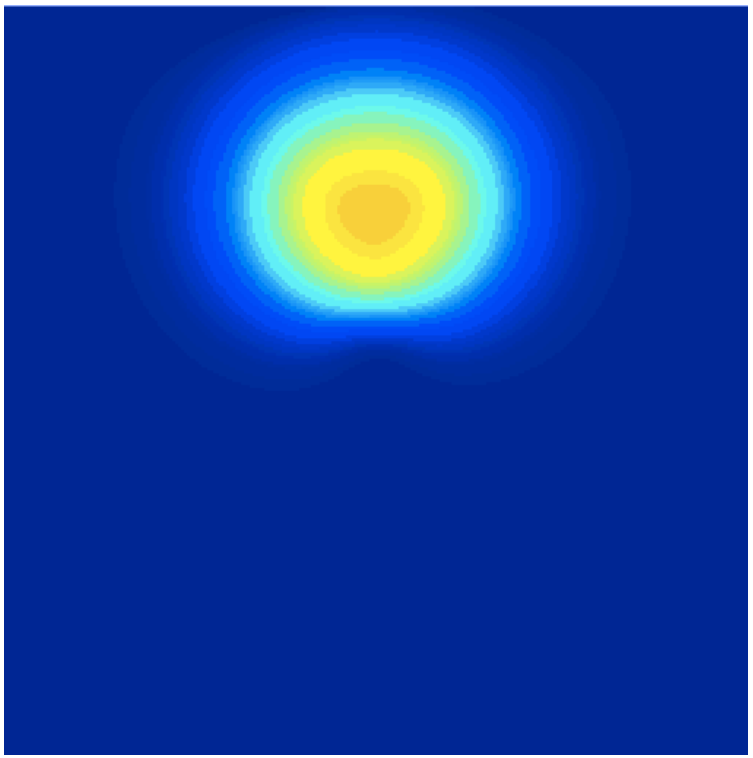


Test of Zalesak [Z79]: 5 revolutions

Random Scheme:

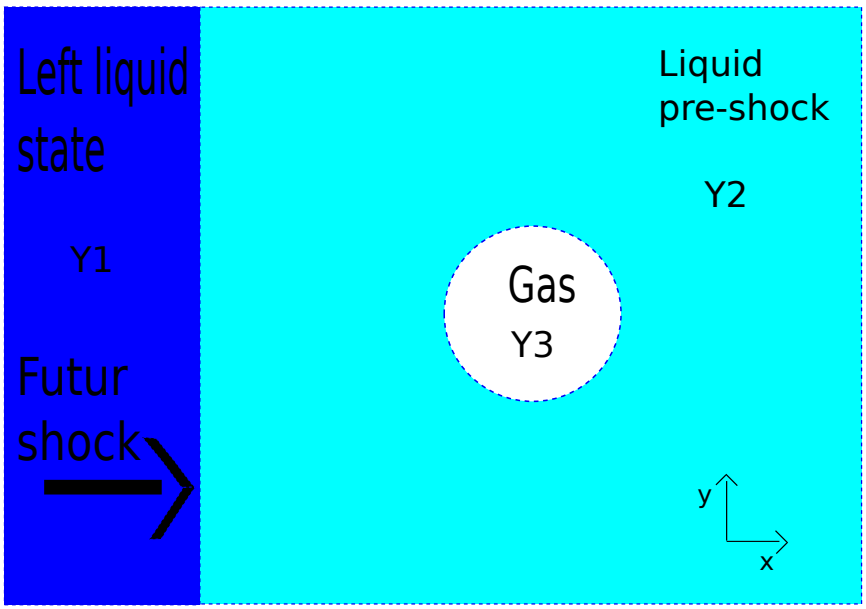


Conservative scheme:



Shock bubble interaction (water/gas)

We consider the shock-bubble interaction test of [KL2010]

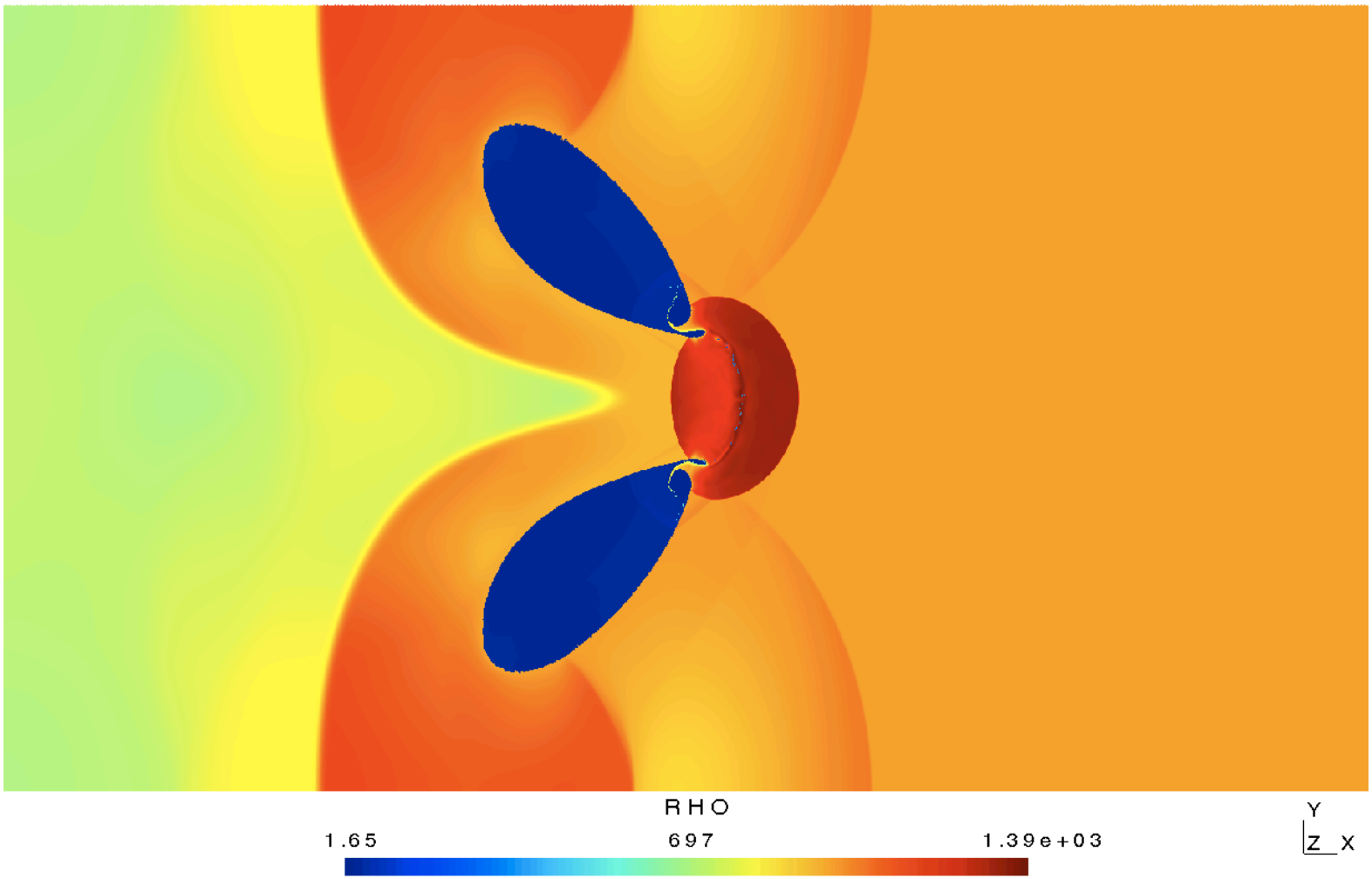


The initial data are:

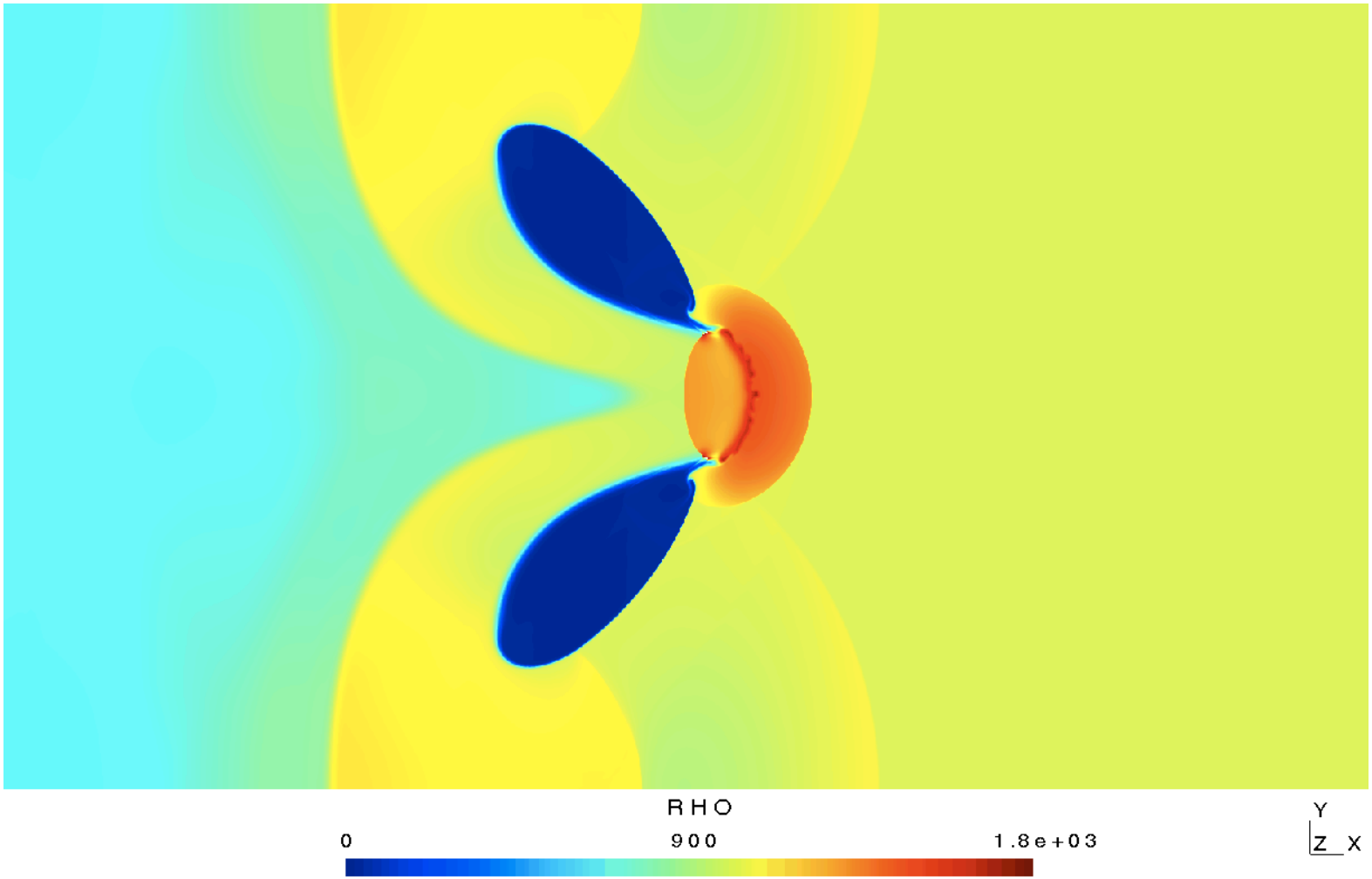
Quantities	Y1	Y2	Y3
$\rho(kg.m^{-3})$	1030.9	1000	1
$u(m.s^{-1})$	300	0	0
$v(m.s^{-1})$	0	0	0
$p(Pa)$	3.0e9	1.0e5	1.0e5
φ	0	0	1
γ	4.4	4.4	1.4
$p_\infty(Pa)$	6.8e8	6.8e8	0

Final time=$450\mu s$.
Grid: $14\,000 \times 7\,000$.
Time of computation on GPU (AMD Radeon HD7970): $\approx 1h$.

Density ρ at $450\mu s$: Random Scheme

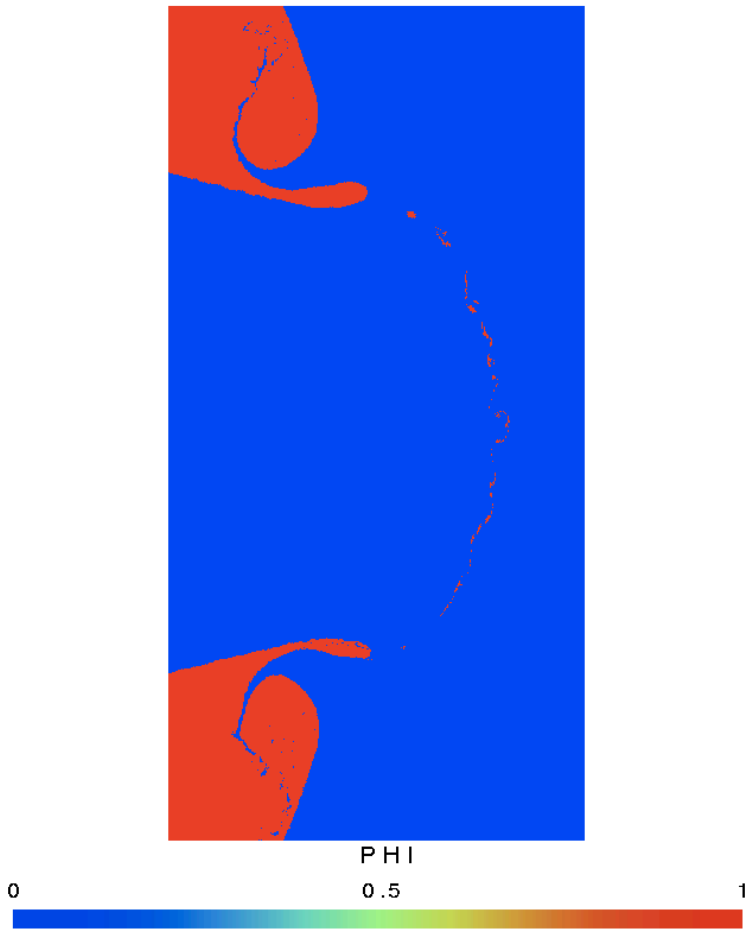


Density ρ at $450\mu s$: NonConsPhi scheme

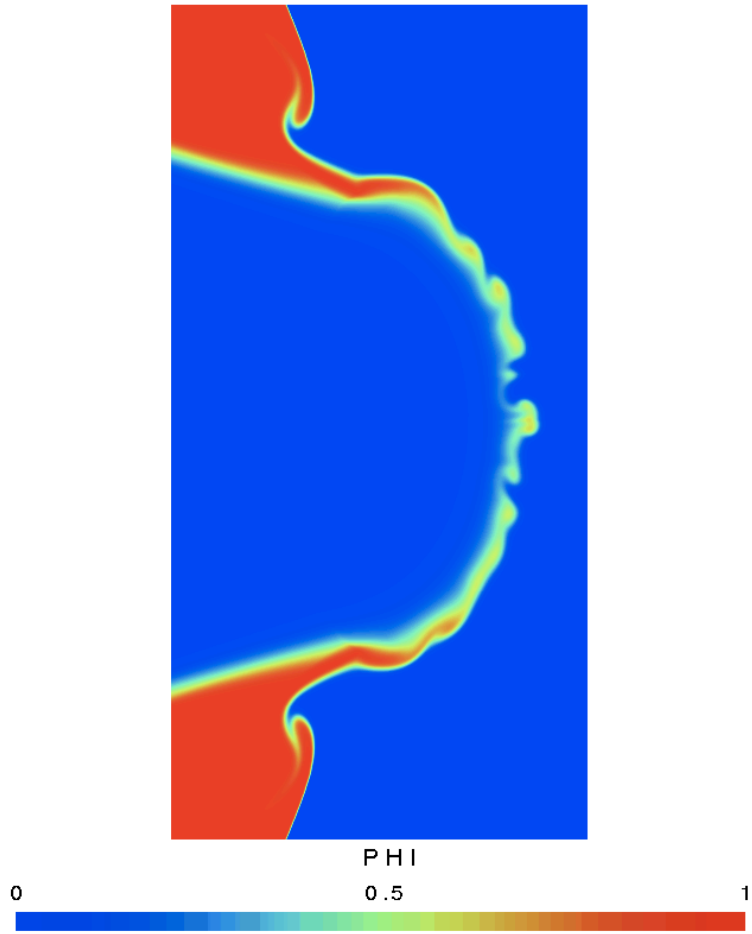


Zoom at $450\mu s$

Random Scheme:



NonConsPhi scheme:



$\rightarrow$ Different shapes with the two algorithms.

GPU and MPI implementation

What is a GPU?

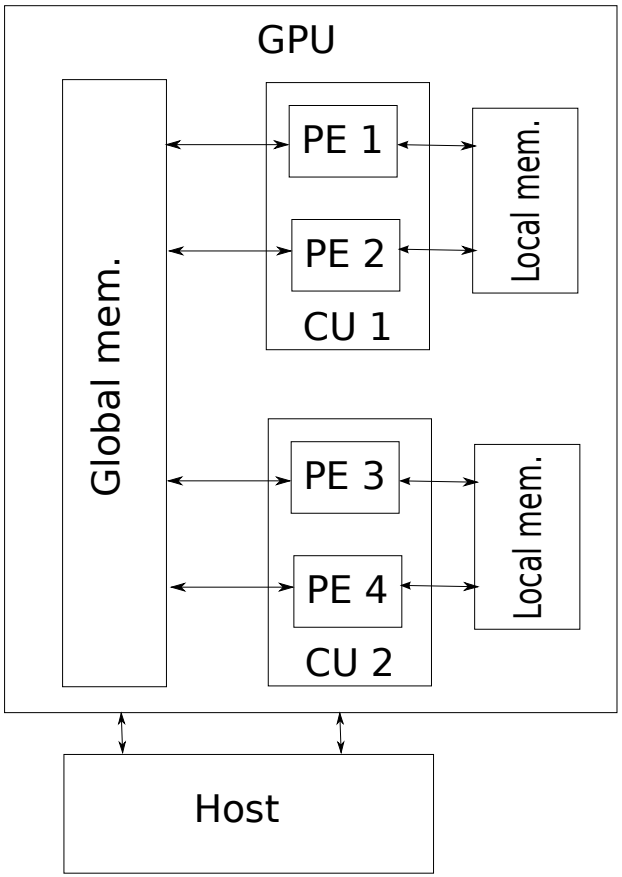
A modern Graphics Processing Unit (GPU) is made of:

- Global memory ($\approx$ 1 GB).
- Compute units ($\approx$ 27).

Each compute unit is made of:

- Processing elements ($\approx$ 8).
- Local memory ($\approx$16 kB).

A GPU with 2 Compute Units and 4 Processing Elements.



Programming rules on GPU

The same program can be executed on all the processing elements at the same time.

- All the processing elements have access to the global memory.
- The processing elements have only access to the local cache memory of their compute unit.
- The access to the global memory is slow while the access to the local memory is fast.
- The access to global memory is much faster if two neighboring processing elements read (or write) into two neighboring memory locations.
  $\rightarrow$ this is called "coalescent memory access".

OpenCL

- OpenCL means "Open Computing Language". It includes:
  - A library of C functions, called from the host, in order to drive the GPU.
  - A C-like language for writing the kernels that will be executed on the processing elements.
- Virtually, it allows to have as many compute units (work-groups) and processing elements (work-items) as needed.
- The threads are sent to the GPU thanks to a mechanism of command queues on the real compute units and processing elements.
- Portable: the same program can run on a multicore CPU and a GPU. It is also possible to manage several devices in the same program.

Implementation in 2D

- Initialization: we initialize the data on the CPU and we send all the data to the global memory of the GPU.
- For each time step:
  - We associate to each row of the grid a work-group and to each cell of the row a virtual processor (work-item). We perform the flux computations and projections in the x direction for each work-group.
  - We "transpose": we exchange the ρu and ρv components and we reorganize the data such that the x and y coordinates are exchanged.
  - We perform the flux computations and projections in the y direction for each work group. Thanks to the transposition, the memory access is coalescent.
  - We transpose to have the correct value in the correct place for the next time step.
- We send all the data to the CPU for post-processing.

Relaxation solver

- $F(W_L, W_R, \xi) = F(R(W_L, W_R, \xi)) - \xi R(W_L, W_R, \xi)$
  $\rightarrow$ to compute the numerical flux, we need to solve the exact Riemann solver.
  $\rightarrow$ It is not efficient on GPU.
- We construct a relaxation solveur $\tilde{R}(W_L, W_R, \xi)$. We extend the Bouchut relaxation solver [B04] to
  - the two-fluid flow,
  - the ALE approach,
  - the projection.
- With the relaxation solver $\tilde{R}$, the Random Scheme keeps the same properties:
  - it preserve the hyperbolic set $\Omega_{\varphi=0} \cup \Omega_{\varphi=1}$,
  - it is statistically conservative and satisfies a discrete entropy inequality,
  - no spurious oscillations,
  - it handles vacuum.

Speedup

Implementation on GPU : The computation corresponds to 300 iterations on a grid 1024×512.

		Computation time (s)	Speedup
AMD A8 3850 (1 coeur)	CPU	527	1
AMD A8 3850 (4 coeurs)	CPU	205	2.6
NVIDIA GeForce 320M	GPU	56	9.4
AMD Radeon HD5850	GPU	3	175
AMD Radeon HD7970	GPU	2	260

There are two points to obtain this speedup:

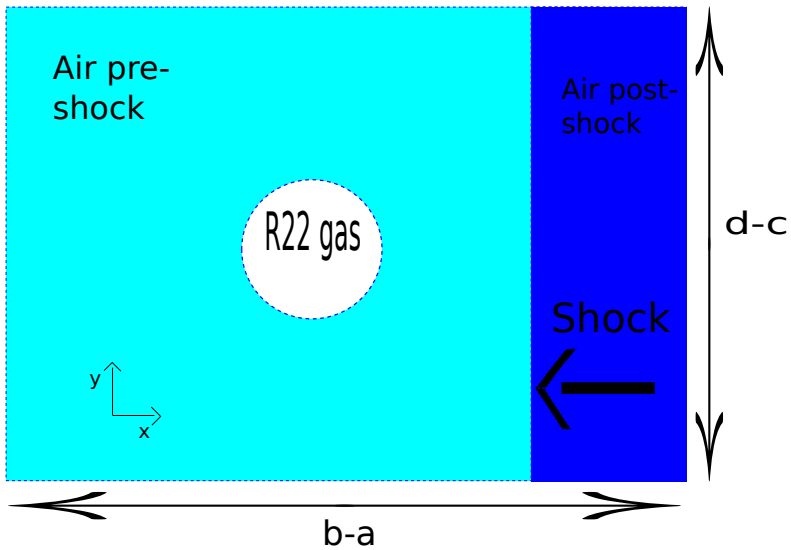
- we used an **optimized transposition** to have coalescent access in x and y directions.
  $\rightarrow$ 10 times faster with the transposition.
- the **relaxation solver**.
  $\rightarrow$ 50 times faster on GPU than exact solver.

OpenGL: video

- The code is very efficient on GPU.
  → The limiting factor is the memory of the GPU (1GB).

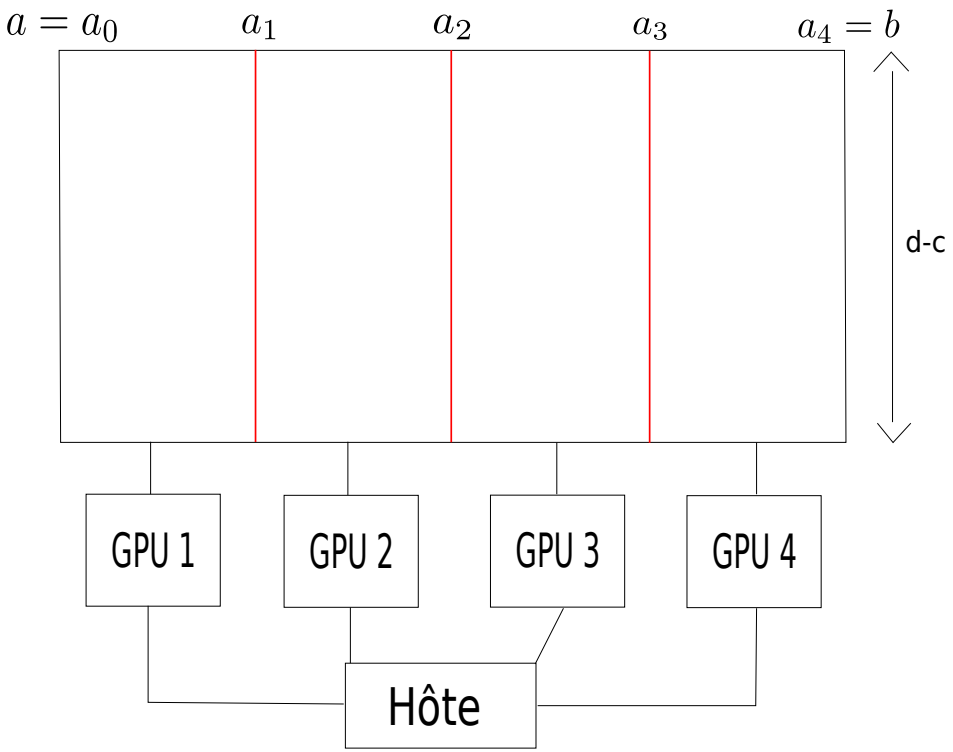
- We introduce Message Passing Interface (MPI) to consider finer mesh.

We present the 4 GPUs MPI implementation for the following test case



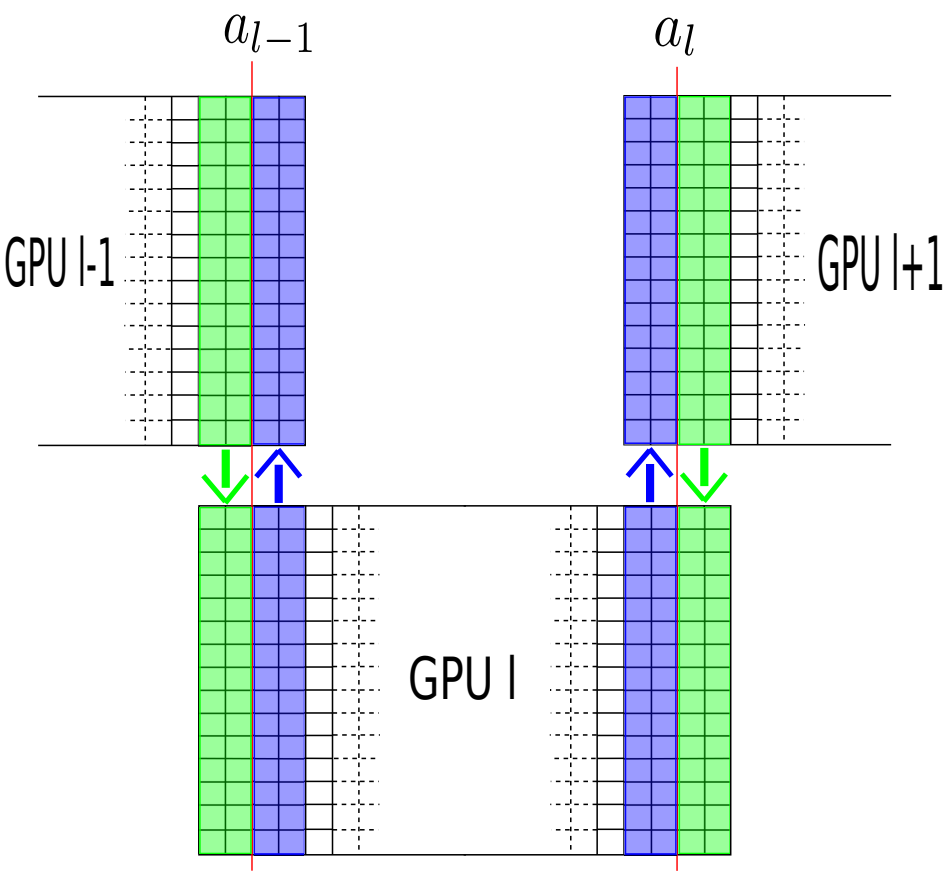
Splitting of the domain

- We split the domain in the x direction into 4 subdomains with a small overlap.
- Each GPU is associated to a subdomain.



MPI communications

Before each iteration, we have to exchange several layers of cells between neighboring GPUs.



Overlap =

- 1 for classical finite volume scheme,
- 2 for ALE-projection scheme,
- 5 for ALE-projection scheme with second order MUSCL reconstruction.

Speedup

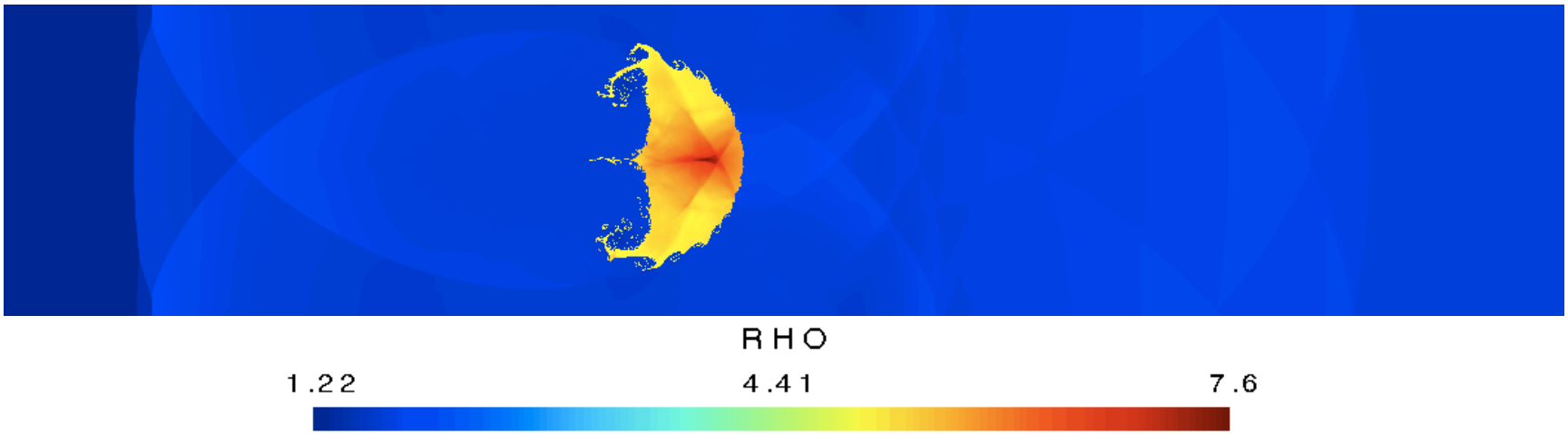
Implementation on MPI : Computation on a cluster of 4 cards AMD Radeon HD7970

Grille	1 GPU	4 GPUs	Speedup
2048×2048	14 s	14 s	1
4096×2048	22 s	16 s	1.4
4096×4096	77 s	60 s	1.3
8192×4096	150 s ?	61 s	2.5
16384×4096	600 s ?	230 s	2.6

$\rightarrow$ We can consider finer meshes. The MPI speedup is > 1 but not optimal. It could be improved by computations and communications overlap.

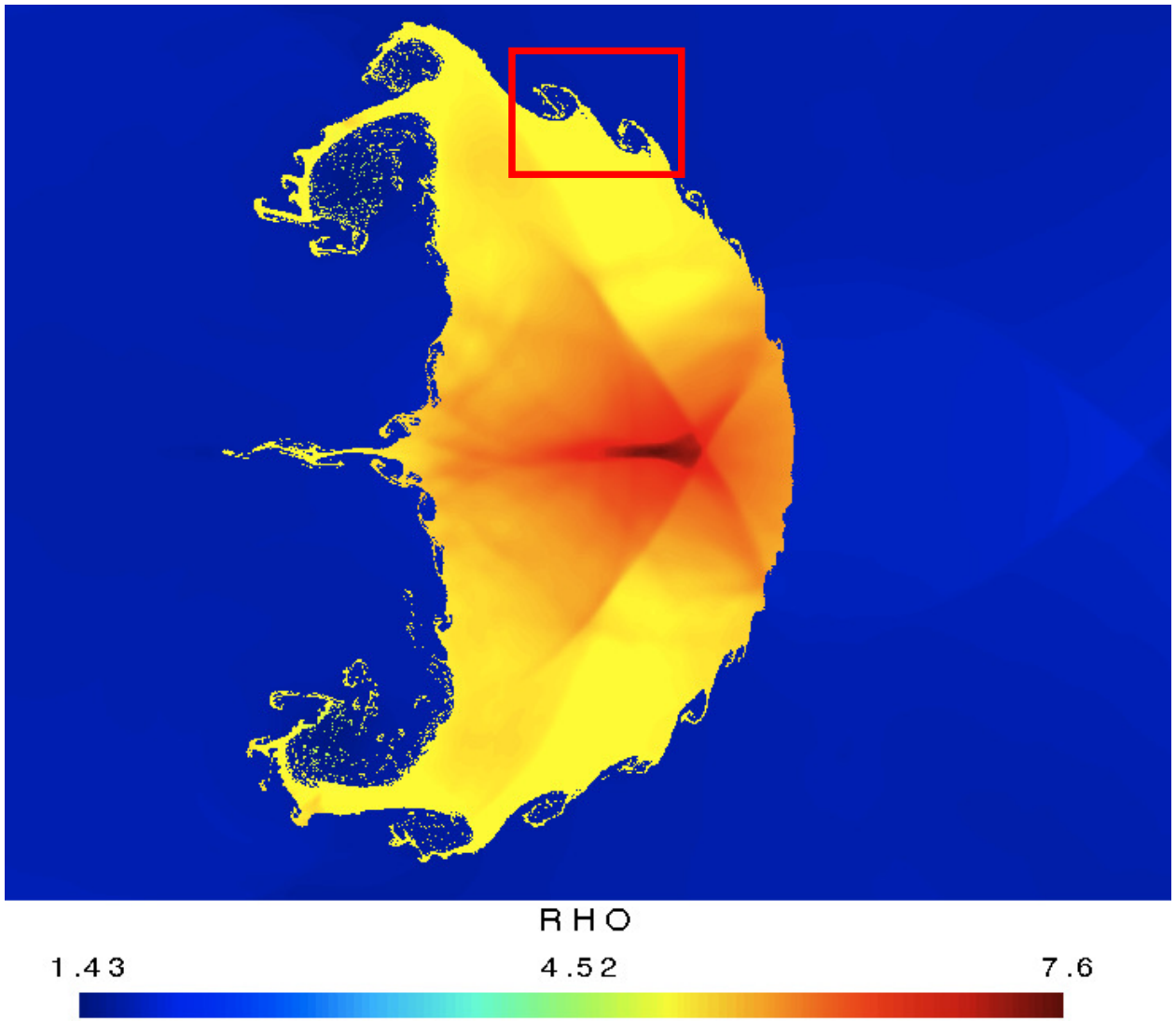
Schock R22/Air interaction: ρ at $600\mu s$

We consider a mesh of $20\,000 \times 5\,000$ cells.

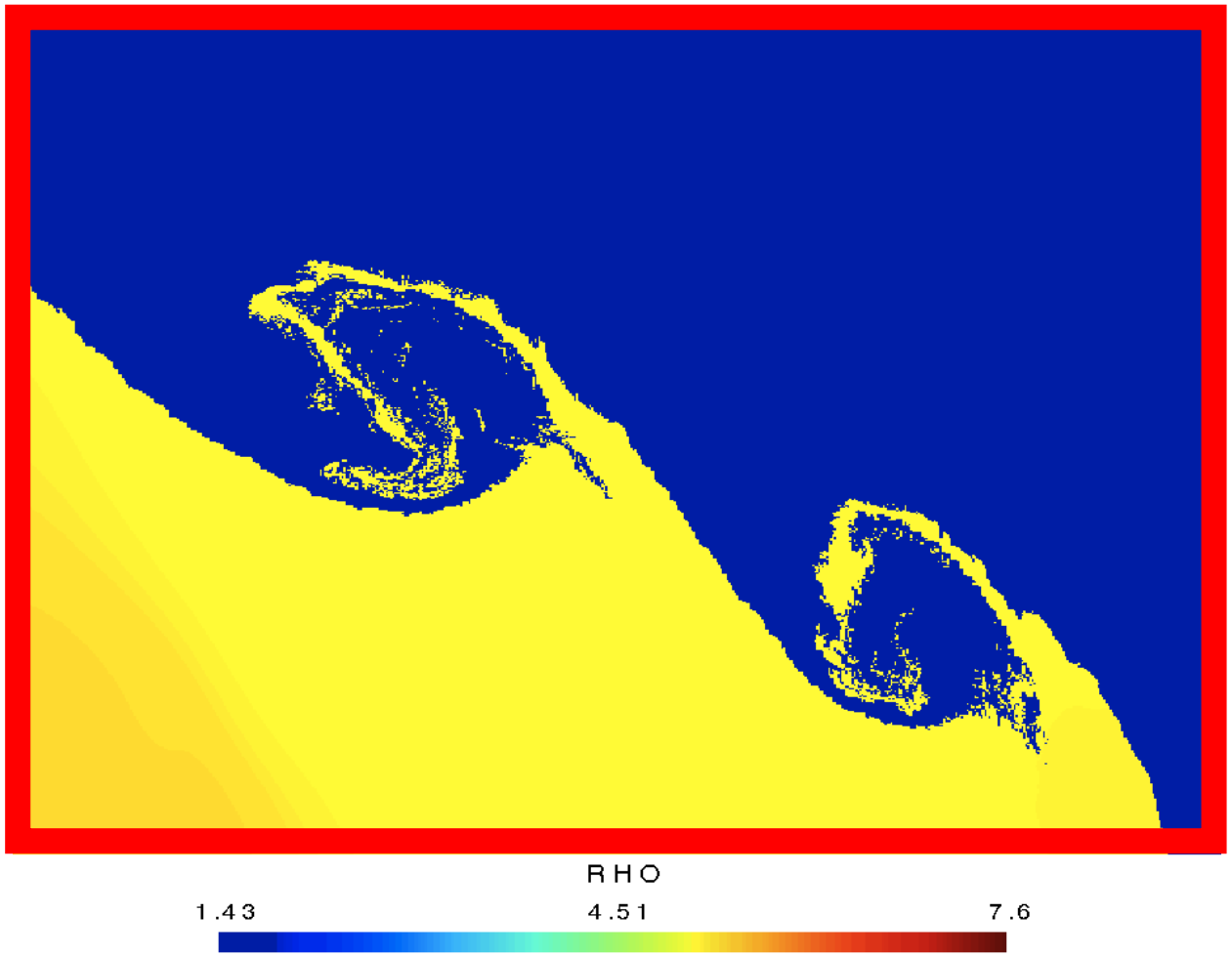


$\rightarrow$ We zoom on the bubble.

Schock R22/Air interaction: zoom on the bubble



Schock R22/Air interaction: zoom on the Rayleigh-Taylor instabilities



Conclusion and perspective

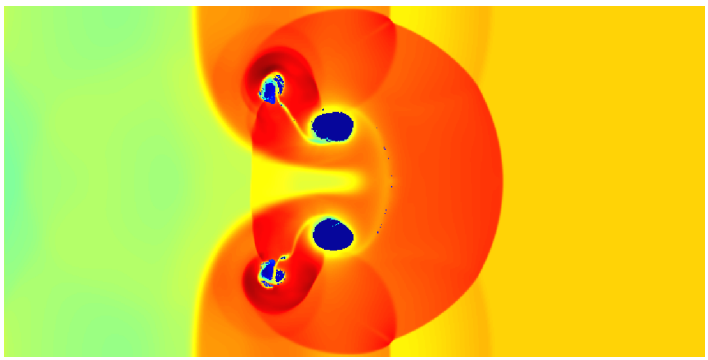
Conclusion:

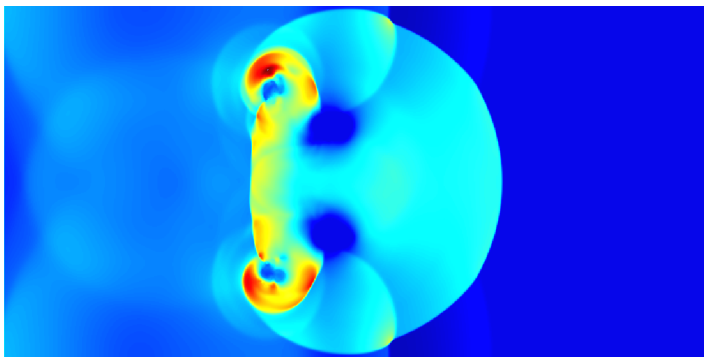
- We developed a robust algorithm to treat complex two-fluid compressible flows. The RS has the following properties:
  - it preserves the hyperbolic set without diffusion $\Omega_0 \cup \Omega_1$,
  - it is statistically conservative and satisfies a discrete entropy inequality.
- The code is very efficient on GPU+MPI, we need only few minutes to compute a complex flow on a mesh with millions of cells.

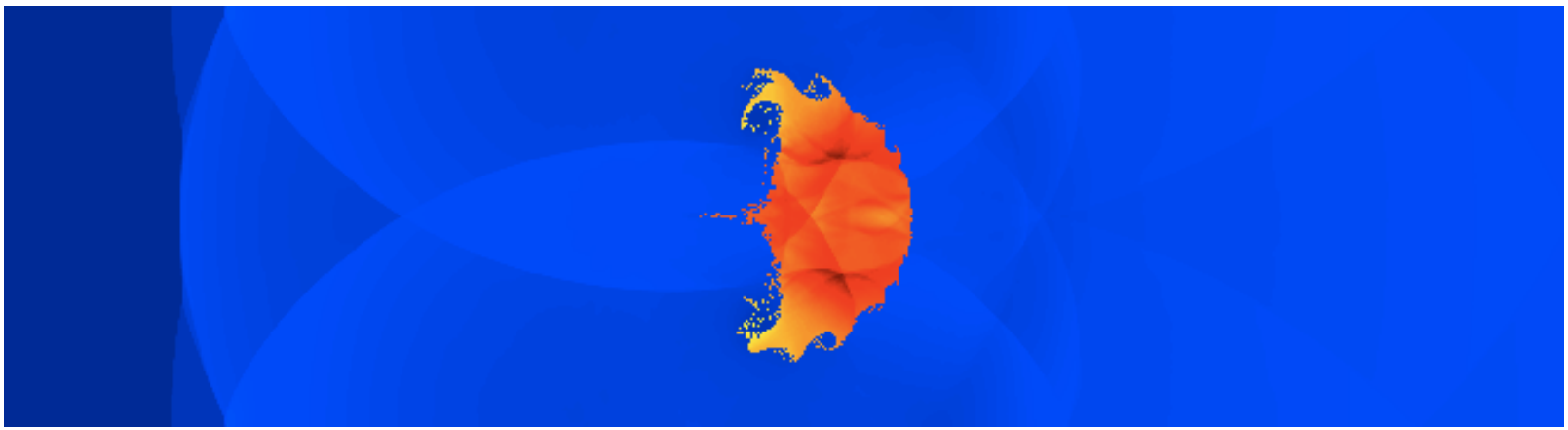
Perspectives:

- Extend the code in 3D.
- Test the Random Scheme with the "hyperbolic convex" pressure law $\tilde{p}$.
- Extend the approach to low Mach number flows.

Thank you for your attention!







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