3. Intégration numérique

3.4 Exercices

Exercice 15

```
> restart;
> intrap := proc(f, a, b, N)
  local h :
  h := (b - a)/N :
  (h*(subs(x = a, f) + subs(x = b, f)))/2 +
  h*add(subs(x = a + k*h, f), k = 1 .. N - 1) :
  end proc:
> intrap(x, 0, 1, 10);
```

$$\frac{1}{2} \qquad \textbf{(1.1.1.1)}$$

```
> intrap(x^2, 0, 1, 10);
```

$$\frac{67}{200} \qquad \textbf{(1.1.1.2)}$$

```
> evalf(intrap(x^2, 0, 1, 100));
```

$$0.3333500000 \qquad \textbf{(1.1.1.3)}$$

```
>
```

Exercice 17

```
> intrecd := proc(f, a, b, N)
  local h :
  h := (b - a)/N :
  h*add(subs(x = a + k*h, f), k = 1 .. N) :
  end proc:
> intrecd(1, 0, 1, 10);
```

$$1 \qquad \textbf{(1.1.2.1)}$$

```
> evalf(intrecd(x, 0, 1, 10));
```

$$0.5500000000 \qquad \textbf{(1.1.2.2)}$$

```
> evalf(intrecd(x, 0, 1, 10000));
```

$$0.5000500000 \qquad \textbf{(1.1.2.3)}$$

```
>
```

Exercice 16

```
> intsimp := proc(f, a, b, N)
  local h:
  h := (b - a)/N:
  add((h/3)·subs(x = a + 2·k·h, f) +
  (4·h/3)·subs(x = a + (2·k + 1)·h, f) +
  (h/3)·subs(x = a + (2·k + 2)·h, f), k = 0 .. N/2 - 1):
  end proc:
```

```
> intsimp(1, 0, 1, 10);
```

$$1 \qquad \textbf{(1.1.3.1)}$$

```
> intsimp(x, 0, 1, 10);
```

$$\frac{1}{2} \qquad \textbf{(1.1.3.2)}$$

```
> intsimp(x^2, 0, 1, 10);
```

$$\frac{1}{3} \qquad \textbf{(1.1.3.3)}$$

```
> intsimp(x^3, 0, 1, 10);
```

$$\frac{1}{4} \qquad \textbf{(1.1.3.4)}$$

```
> evalf(intsimp(x^4, 0, 1, 100));
```

$$0.2000000013 \qquad \textbf{(1.1.3.5)}$$

```
> evalf(1/5);
```

$$0.2000000000 \qquad \textbf{(1.1.3.6)}$$

```
>
```

Exercice 20

```
> V := [0, 19.5, 35, 45, 40.5, 25, 20.5, 29, 27, 12.5, 0];
```

$$V := [0, 19.5, 35, 45, 40.5, 25, 20.5, 29, 27, 12.5, 0] \qquad \textbf{(1.1.4.1)}$$

```
> f := x→x^2:
  V2 := map(f, V);
```

$$V2 := [0, 380.25, 1225, 2025, 1640.25, 625, 420.25, 841, 729, 156.25, 0] \qquad \textbf{(1.1.4.2)}$$

```
> N := 10:
  h := (0.1 - 0)/N:
  s := add((h/3)·V2[k + 1] +
```

$$\frac{4\cdot h}{3}\cdot V2[k+2]+\frac{h}{3}\cdot V2[k+3], k=0..\frac{N}{2}-1\Big);$$

$$s := 115.9600000 \quad \textbf{(1.1.4.3)}$$

```
> Vrms := sqrt( 1/(0.1 - 0) * s );
```

$$Vrms := 34.05290002 \quad \textbf{(1.1.4.4)}$$

```
>
```

5. Equatios différentielles

5.4 Implémentation

```
> #Méthode d'Euler explicite
> euler2 := proc( f, y0, t0, T, N )
  local h, y, n :
  h := (T - t0)/N :
  y := array(0..N) :
  y[0] := y0;
  for n from 1 to N - 1 do
  y[n + 1] := y[n] + h*f(t0 + n*h, y[n]) :
  od:
  end proc:
>
> #Méthode de Runge
> runge := proc( f, y0, t0, T, N )
  local h, y, k1, k2, n :
  h := (T - t0)/N :
  y := array(0..N) :
  y[0] := y0 :
  for n from 1 to N - 1 do
  k1 := f(t0 + n*h, y[n]) :
  k2 := f(t0 + n*h, y[n] + h/2*k1) :
  y[n + 1] := y[n] + h*k2 :
  od:
  end proc:
>
> #Méthode de Runge-Kutta d'ordre 4
```

```
> rk4 := proc( f, y0, t0, T, N )
  local h, y, k1, k2, k3, k4, n :
  h := (T - t0)/N :
  y := array(0..N) :
  y[0] := y0 :
  for n from 1 to N - 1 do
  k1 := f(t0 + n·h, y[n]) :
  k2 := f(t0 + n·h + h/2, y[n] + h/2·k1) :
  k3 := f(t0 + n·h + h/2, y[n] + h/2·k2) :
  k4 := f(t0 + (n + 1)·h, y[n] + h·k3) :
  y[n + 1] := y[n] + h·(k1 + 2·k2 + 2·k3 + k4)/6 :
  od:
  end proc:
>
```