Quantitative Risk Analysis
Master of Financial Engineering - M2
Intermediate Exam - Duration : 1 hour
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No documents - You can use a calculator

Please report clear and detailed answers explicitly, as well as all the formulas you use and all the theorems you invoke.

Problem 1 :
1) Give the mathematical definition of VaR_α and explain its meaning.
2) Give the mathematical definition of CVaR_α and explain its meaning.

Problem 2 : We consider an investment for which the price at time $t = 0$ is 100 euros and the value at time $T > 0$ is a random variable V. The density of the random variable V is given by :

$$\begin{cases} P(V = 80) = 0.01, \\ P(V = 90) = 0.03, \\ P(V = 95) = 0.26, \\ P(V = 100) = 0.30, \\ P(V = 105) = 0.10, \\ P(V = 106) = 0.30. \end{cases}$$

1) Write the loss X at time T as a function of V.
2) Give the density of the random variable X.
3) Compute $\text{VaR}_{0.9}(X)$. Explain your method.
4) What is the biggest value α such that $\text{VaR}_\alpha(X) = 0$?

Problem 3 : Consider two independent securities Y_1 and Y_2. At time $t = 0$, these two securities cost 20 euros. At time $t = T$, the value of security $i = 2$ is exponentially distributed with an average of 20 euros. At time $t = T$ the value of security $i = 1$ is 22 with a probability of 90% and 15 with a probability of 10%. We recall that the density of the exponential distribution with average μ is

$$f(x) = \begin{cases} \frac{1}{\mu} e^{-\frac{x}{\mu}}, & \text{if } x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

1) Give the density of the first security $i = 1$.
2) Compute $VaR_{0.95}$ for the loss at time $t = T$ of security $i = 1$.
3) Compute the cumulative function of an exponential random variable with average μ.
4) Compute $VaR_{0.95}$ for the loss at time $t = T$ of security $i = 2$.
5) Compute the $0.95 - VaR$ at time $t = T$ of a portfolio made up of these two securities.
6) Give the definition of a sub-additive risk measure ? Can VaR be a sub-additive variable ?