Quantitative Risk Analysis
Master of Financial Engineering - M2
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Fourth Practical : Multivariate Case

Consider two independent random variables X and Y following a standard normal law. Let Z be the multivariate random variable defined as :

$$Z = (X, Y).$$

1) Using Matlab function surf, plot the joint density $f(x, y)$ of Z as a surface. Hint : a possible equation of this surface is $z = f(x, y)$.
2) Using the Box-Miller random number generator, write a Matlab function that generates n realizations of random variable Z and that plots these realizations in a two-dimensional graph.
3) For any positive number r, call $h(x, y) = \mathbb{1}_{[r,+\infty[}(\sqrt{x^2 + y^2})$. Plot the function $h(x, y).f(x, y)$ as a surface using the Matlab function surf.

Assume that S_t is the price of some security at time $t = 0, 1, ..., T$. Further assume that this security follows a geometric brownian motion of drift r (equal to the risk-free rate) and volatility σ^2. Recall that, in this case :

$$S_{t+1} = S_t e^{(r-\sigma^2/2)\frac{1}{T}+\sigma X_{t+1}},$$

where X_{t+1} is a standard normal random variable.

4) Express random variable S_t as a function of X_1, ..., X_t, and of the price S_0 of the security at time $t = 0$.
5) Now consider an asian call option on this security with strike K and expiration date T. The price of this call is $C = E_f(h(X_1, ..., X_T))$, where f is the joint density distribution of $(X_1, ..., X_T)$. Give the expressions of f and h.
6) Write a Matlab function that evaluates C using the Monte Carlo method. The arguments of this function will be T, K, S_0, r, σ, and the number M

of simulations to perform.

7) Implement error control in the function designed in question 6), adding ε and confidence level p as new arguments of your function, and removing M. Use the sequential method for error control.
8) Implement importance sampling in the function designed in question 7). Density $g(x_1, ..., x_T)$ will equal to $f(x_1, ..., x_T)$ except for its average $(\mu_1, ..., \mu_T)$ that will be a new argument of the function.
9) By trial and error, try to find a value of $(\mu_1, ..., \mu_T)$ that reduces the variance of your estimator. For the tests, choose : $S_0 = 100$, $K = 150$, $T = 100$, $r = 2\%$, $\sigma = 0.5$, $\varepsilon = 1\%$, $p = 95\%$.