Advanced Algorithms for Finance
Master of Financial Engineering - M2
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Fourth Practical : random variable generation

1) Design a Matlab function that takes the three parameters a, c, and m of the congruential generator as arguments as well as the i-th term y_i of the pseudo-random sequence, and that returns y_{i+1}.
2) What is the value of y_{10} when $m = 2^{31} - 1$, $a = 16807$, and $c = 0$ with $y_0 = 1$? *Remark : this generator, proposeed by Park and Miller is a good generator, properly tested, and called the Standard Minimal).*
3) Design a Matlab function that takes as an imput two sets of parameters (a_x, c_x, m_x) and (a_y, c_y, m_y) of the congruential generator, two seeds x_0 and y_0 and two integers k and N, and that returns the vector of the k first values of the sequence z_i obtained with the mixing method.

We now want to test our generators. To this end, we will use two kind of tests : the spectral test that consists in plotting the pairs $(y_{2i}/m, y_{2i+1}/m)$ within the square $[0, 1[\times[0, 1[$, and the adequacy test that consists in plotting the cumulative function of the obtained random numbers.

4) Plot pairs $(y_{2i}/m, y_{2i+1}/m)$ using 10000 numbers from the following generators with $y_0 = 1$:
   i. $m = 2^{31} - 1$, $a = 16807$, $c = 0$ (Standard Minimal),
   ii. $m = 2^{31}$, $a = 65539$, $c = 0$ (RANDU implemented in the IBM System/310),
   iii. $m = 2^{32}$, $a = 129$, $c = 907633385$ (the generator of Turbo Pascal),
   iv. $m = 2^{31} - 1$, $a = 127$, $c = 0$,
   v. $m = 2^{31} - 1$, $a = 31$, $c = 0$,
   vi. $m = 2^{29} - 1$, $a = 3$, $c = 0$.
   vii. $m = 2^{48}$, $a = 31167285$, $c = 1$ (the generator of Lavaux and Jenssens).

What do you observe ? Which of these are good generators ?

5) Does the mixing method provide a significant improvement of the quality of pseudo-random sequences, measured with the spectral test ?
6) Write a function that plots an estimate of the cumulative function of a randor number generator and another function that plots its density function. Plot these estimates for the seven generators given above (with 10000 terms of each sequence).

The few next questions aim at building a generator for the normal distribution.

7) Explain why rejection sampling cannot be used to generate realizations of a normal distributions when $g(x)$ is the density of a uniform distribution.
8) Plot the density $f(x)$ of the standard normal law as well as the following density :
$$g(x) = \frac{1}{2}e^{-|x|}.$$
Find a value of M so that $f(x) \leq Mg(x)$.
9) Write a Matlab function that takes the three parameters a, c, and m of the congruential generator as arguments as well as two values μ and σ, a seed y_0 and an integer k and that returns k realisations of the standard normal law obtained by rejection sampling using density g and the value of M from previous question. *Hint : The realizations following density $g(x)$ will be found using inverse transform sampling.*
10) Write a Matlab function that takes the three parameters a, c, and m of the congruential generator as arguments as well as two values μ and σ, a seed y_0 and an integer k and that returns k realisations of the standard normal law obtained by the Box and Miller method.
11) Using Question 6), plot the density of the standard normal distribution obtained with the rejection sampling and the Box and Miler method.

The remaining questions are about Monte-Carlo simulation.

12) In a television show, the announcer shows three closed doors to the candidate. Behind one of these three doors is found a prize and the candidate must choose a door to open. When the candidate has chosen, the announcer does not open the chosen door, but one of the other two, behind which there is no

prize. The candidate can then maintain his/her choice, or change it. What is your advice to him/her ?

13) Use the Monte-Carlo method to evaluate the probability to get the prize under the assumption that the candidate does not modify or modifies his/her choice (use the generator of Lavaux and Jenssens with $y_0 = 1$ and $M = 100000$ simulations). What is your conclusion ?

14) Consider the pricing model designed during the second practical and write a european option pricer (call and put) using Monte-Carlo simulation. The arguments of the pricer will be the values of r, u, d, K, S, and T, as well as the parameters of the pseudo-random number generator a, c, m, and y_0, and the number M of simulations to carry out.

15) Taking $r = 0.02$, $K = 105$, $u = 1.05$, $d = 0.95$, $S = 100$, give the value of a european put with expiration dates $T = 10$, $T = 20$, and $T = 30$. Compare the results with the results obtained in the Third Practical.

16) With the same underlying security than in question 14), write an asian option pricer (call and put) using Monte-Carlo simulations. The arguments of the pricer will be the values of r, u, d, K, S, and T, as well as the parameters of the pseudo-random number generator a, c, m, and y_0, and the number M of simulations to carry out.

17) Using $r = 0.02$, $K = 105$, $u = 1.05$, $d = 0.95$, $S = 100$, give the value of an asian put with expiration dates $T = 10$, $T = 20$, and $T = 30$.