Computing two compressible fluids flow on GPU

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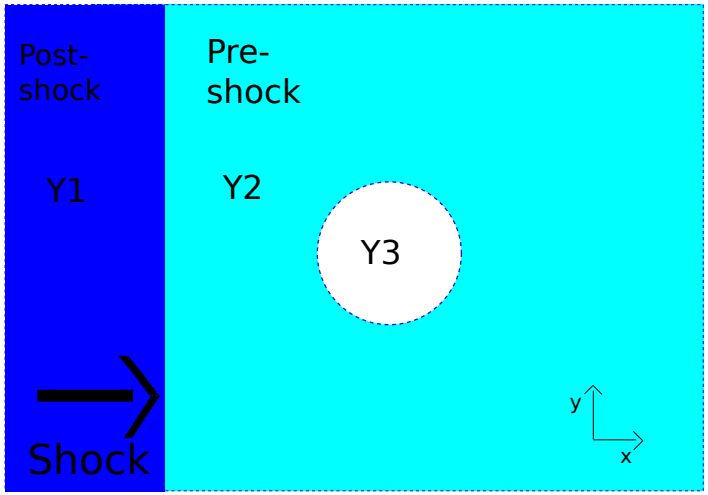
Example of computation :

Study case : a shock wave in the liquid impinging a gas bubble.



Properties of the flow :

- a flow of two immiscible fluids,
- the gas and the water are two compressible fluids.

Plan

Model

We consider the 2D compressible Euler equations :

$$\begin{aligned}
\partial_t(\rho) + \partial_x(\rho u) + \partial_y(\rho v) &= 0, \\
\partial_t(\rho u) + \partial_x(\rho u^2 + p) + \partial_y(\rho uv) &= 0, \\
\partial_t(\rho v) + \partial_x(\rho uv) + \partial_y(\rho v^2 + p) &= 0, \\
\partial_t(\rho E) + \partial_x((\rho E + p)u) + \partial_y((\rho E + p)v) &= 0, \\
\partial_t(\rho \varphi) + \partial_x(\rho u \varphi) + \partial_y(\rho v \varphi) &= 0,
\end{aligned}$$

where ρ is the density, (u, v) the velocity vector, E the total energy and φ the gas mass fraction.

$\Rightarrow$ We obtain a single system for the two fluids.
We have 5 equations for 6 unknowns.

Equation of states

In order to close the system, we assume that p satisfies a stiffened gas pressure law :

$$p(\rho, e, \varphi) = (\gamma(\varphi) - 1)\rho e - \gamma(\varphi)\pi(\varphi),$$

where $e = E - (u^2 + v^2)/2$.
The speed of sound c is defined by $c^2 = \frac{\partial p}{\partial \rho} + \frac{p}{\rho^2}\frac{\partial p}{\partial e}$, then

$$c = \sqrt{\gamma \frac{p + \pi}{\rho}}.$$

For standard pressure and temperature,

- in the gas ($\rho = 1.225$), we have $\varphi = 1$, $\gamma_{gaz} = 1.4$, $c_{gaz} \approx 340 m/s$, then $\pi_{gaz} \approx 0$ (perfect gas law).
- in the liquid ($\rho = 1000$), we have $\varphi = 0$, $\gamma_{liq} = 4.4$, $c_{liq} \approx 1500 m/s$, then $\pi_{liq} \approx 5 \times 10^8$.

Equation of states

- We have

$$\partial_t \varphi + u\partial_x \varphi + v\partial_y \varphi = 0$$

$\Rightarrow$ theoricaly, if φ takes the values 0 or 1 at initial time, it is true at any time (there is not mixture).

- Numerically, we have diffusion. In the numerical mixture we take [SA1999]

$$\begin{aligned} \frac{1}{\gamma(\varphi)-1} &= \varphi\frac{1}{\gamma_{gaz}-1} + (1-\varphi)\frac{1}{\gamma_{liq}-1}, \\ \frac{\gamma(\varphi)\pi(\varphi)}{\gamma(\varphi)-1} &= \varphi\frac{\gamma_{gaz}\pi_{gaz}}{\gamma_{gaz}-1} + (1-\varphi)\frac{\gamma_{liq}\pi_{liq}}{\gamma_{liq}-1}. \end{aligned}$$

Conservative form

We can write the system under the conservative form :

$$\partial_t W + \partial_x F(W) + \partial_y G(W) = 0,$$

where the vector of conservative variables is :

$$W = (\rho, \rho u, \rho v, \rho E, \rho \varphi)^T,$$

and the conservative fluxes are :

$$F(W) = (\rho u, \rho u^2 + p, \rho uv, (\rho E + p)u, \rho \varphi u)^T,$$
$$G(W) = (\rho v, \rho uv, \rho v^2 + p, (\rho E + p)v, \rho \varphi v)^T.$$

Physical properties :

- the density ρ is non negative,
- the internal energie e is non negative,
- the gas mass fraction φ is in $[0; 1]$.

Mathematical properties :

- Under the physical properties, the system is hyperbolic in each direction. For each $n = (n_1, n_2)^T$, the matrix $n_1 D_W F(W) + n_2 D_W G(W)$ is diagonalizable and admits the eigenvalues :

$$\lambda_1 = (u, v)^T.n - c, \quad \lambda_2 = \lambda_3 = \lambda_4 = u, \quad \lambda_5 = (u, v)^T.n + c.$$

- We can show that the Riemann problem admits a unique global entropy solution.
- The pressure and the velocity are continuous at the two-fluid interface.

Aims

We want build a numerical scheme that :

- preserves the non negativity of ρ and e,
- converges (numerically) to the entropy solution,
- introduces no fluctuations on u and p at the two-fluid interface,
- handles vacuum,
- allows an efficient parallelism.

Splitting

In order to solve the 2D equations :

$$\partial_t W + \partial_x F(W) + \partial_y G(W) = 0,$$

we consider a dimensional splitting.
Thus, we only have to explain how to solve the 1D system

$$\partial_t W + \partial_x F(W) = 0,$$

where

$$\begin{aligned} W &= (\rho, \rho u, \rho v, \rho E, \rho \varphi)^T. \\ F(W) &= (\rho u, \rho u^2 + p, \rho uv, (\rho E + p)u, \rho \varphi u)^T. \end{aligned}$$

As we know exact solutions in 1D, we can validate our scheme.

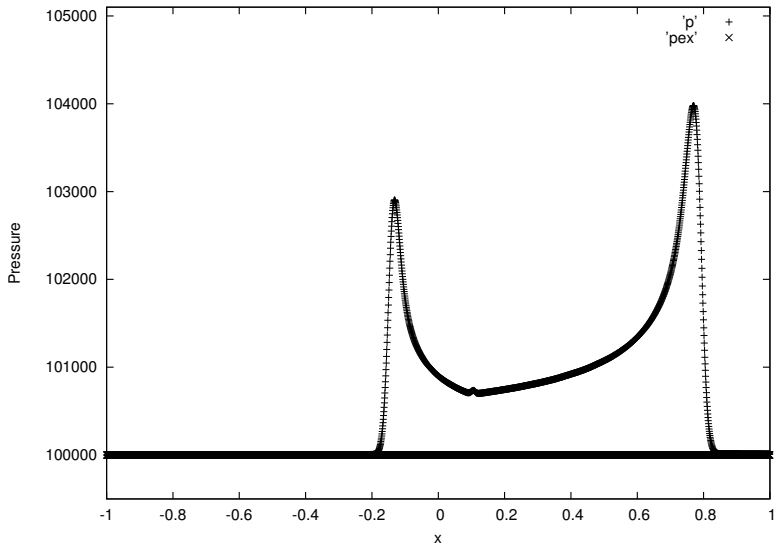
Problem of the classical conservative numerical scheme

We observe oscillations on the pressure at the interface of the two fluids.

If we consider the following Riemann problem :

Quantities	$x < 0$	$x > 0$
$\rho(kg.m^{-3})$	10	1
$u(m.s^{-1})$	50	50
$p(Pa)$	1e5	1e5
φ	1	0
γ	1.4	1.1
π	0	0

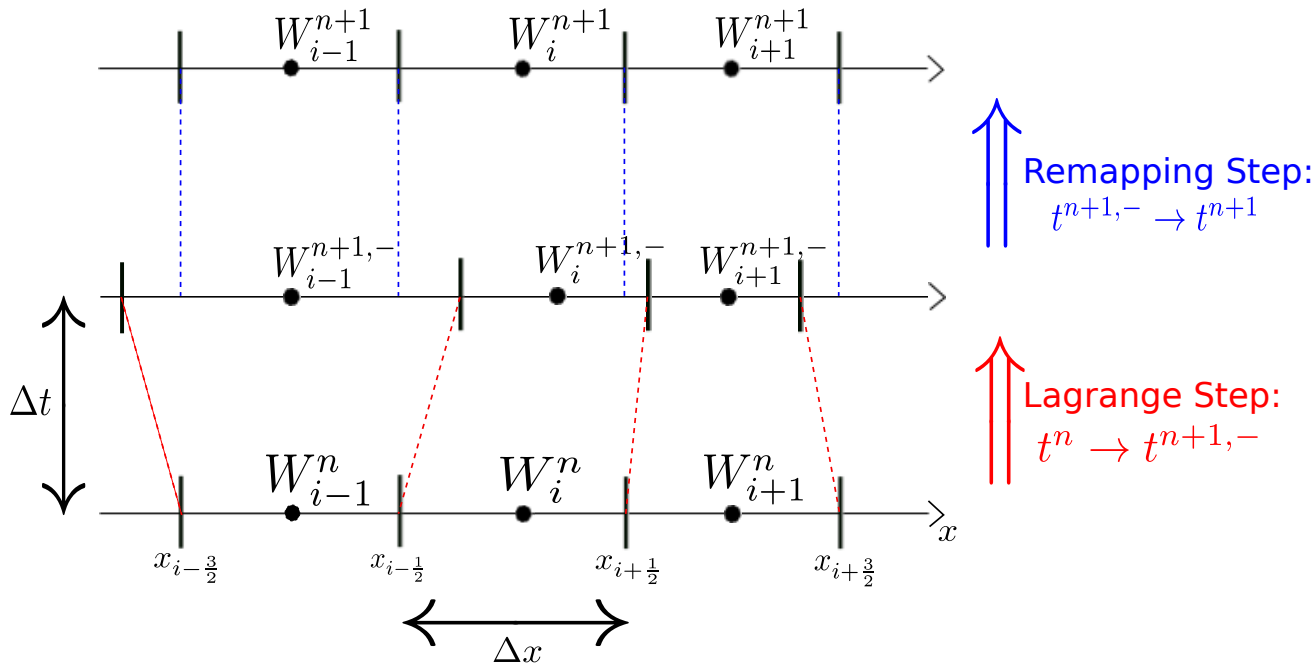
we obtain :



Structure of the random numerical scheme

The scheme includes two steps :

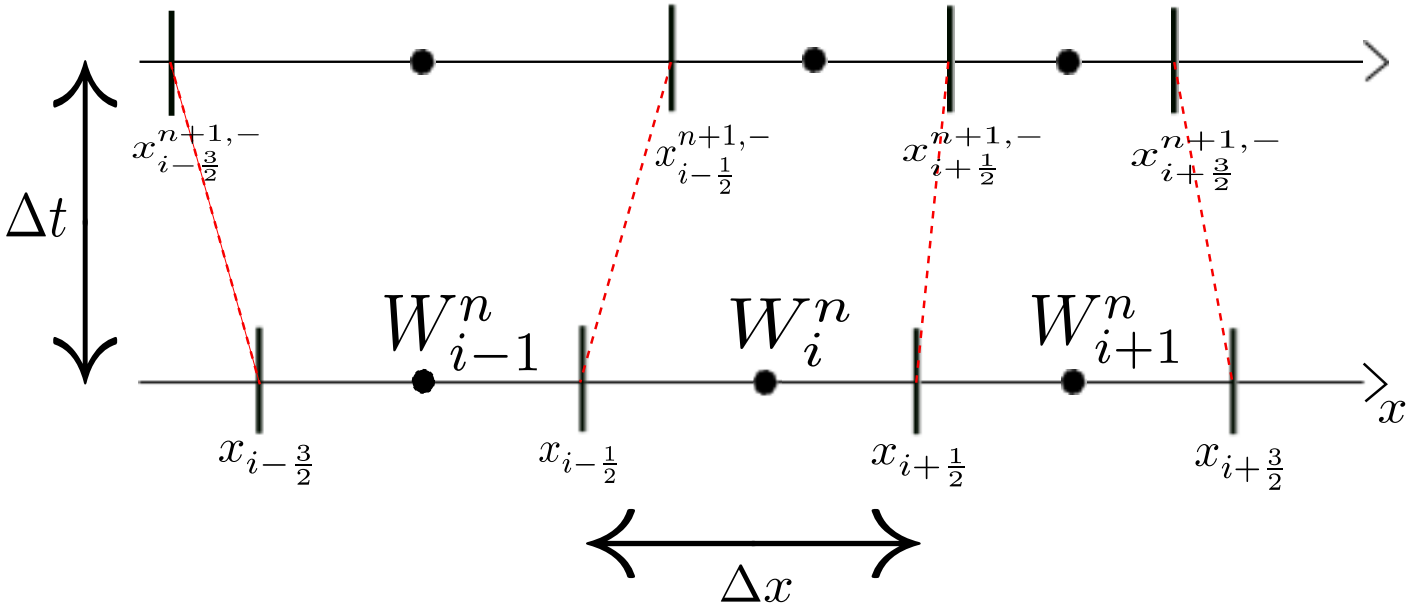
- the Lagrangian step for solving the system $\partial_t W + \partial_x F(W) = 0$, between time t^n and $t^{n+1,-}$ on a moving mesh,
- the remapping step for returning to the initial Eulerian mesh at time t^{n+1}.



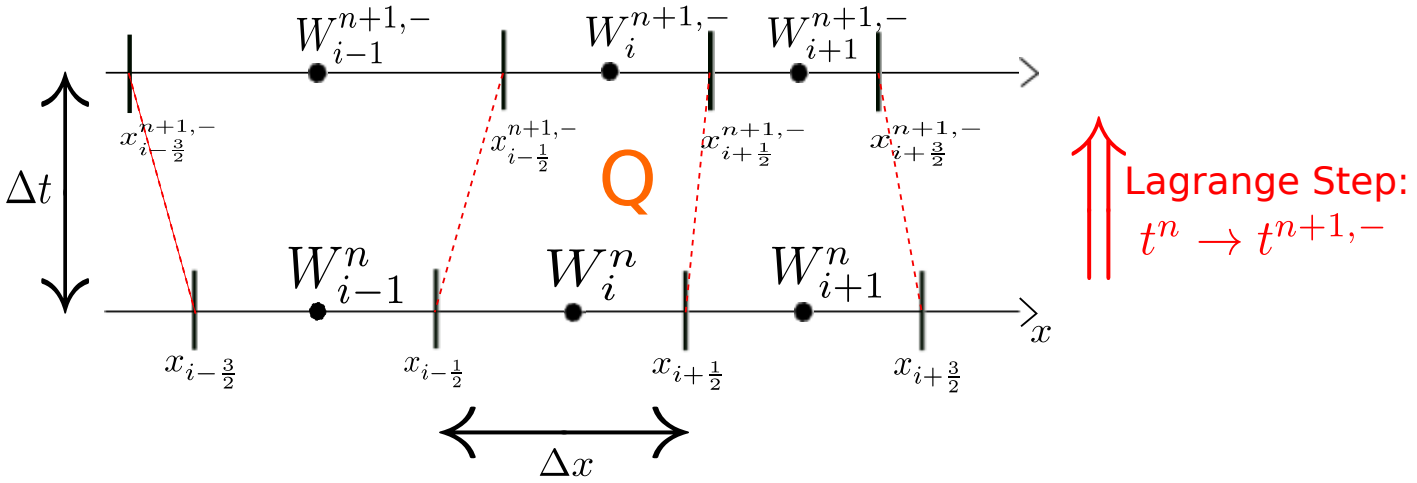
Some notations

- We propose a first order finite volume scheme with a Lagrangian approach, the boundary $x_{i+\frac{1}{2}}$ moves at the velocity of the fluid $u^n_{i+\frac{1}{2}}$ between t^n and $t^{n+1,-}$:

$$x^{n+1,-}_{i+\frac{1}{2}} = x^n_{i+\frac{1}{2}} + \Delta t \, u^n_{i+\frac{1}{2}}.$$



Finite volume scheme



The integration of $\partial_t W + \partial_x F(W) = 0$ on the space-time quadrilateral Q gives :

$$\Delta x_i^{n+1,-} W_i^{n+1,-} - \Delta x W_i^n + \Delta t \left(F(W_i^n, W_{i+1}^n) - F(W_{i-1}^n, W_i^n) \right) = 0$$

where $F(W_L, W_R)$ is the Lagrangian flux and Δt satisfies the CFL condition.

Lagrangian flux

We recall that the Lagrangian flux is :

$$F(W_i^n, W_{i+1}^n) : \quad = \quad (0, p_{i+\frac{1}{2}}^n, 0, u_{i+\frac{1}{2}}^n p_{i+\frac{1}{2}}^n, 0)^T.$$

In order to compute the quantities $i+1/2$, we use a relaxation Riemann solver based on the work of Bouchut :

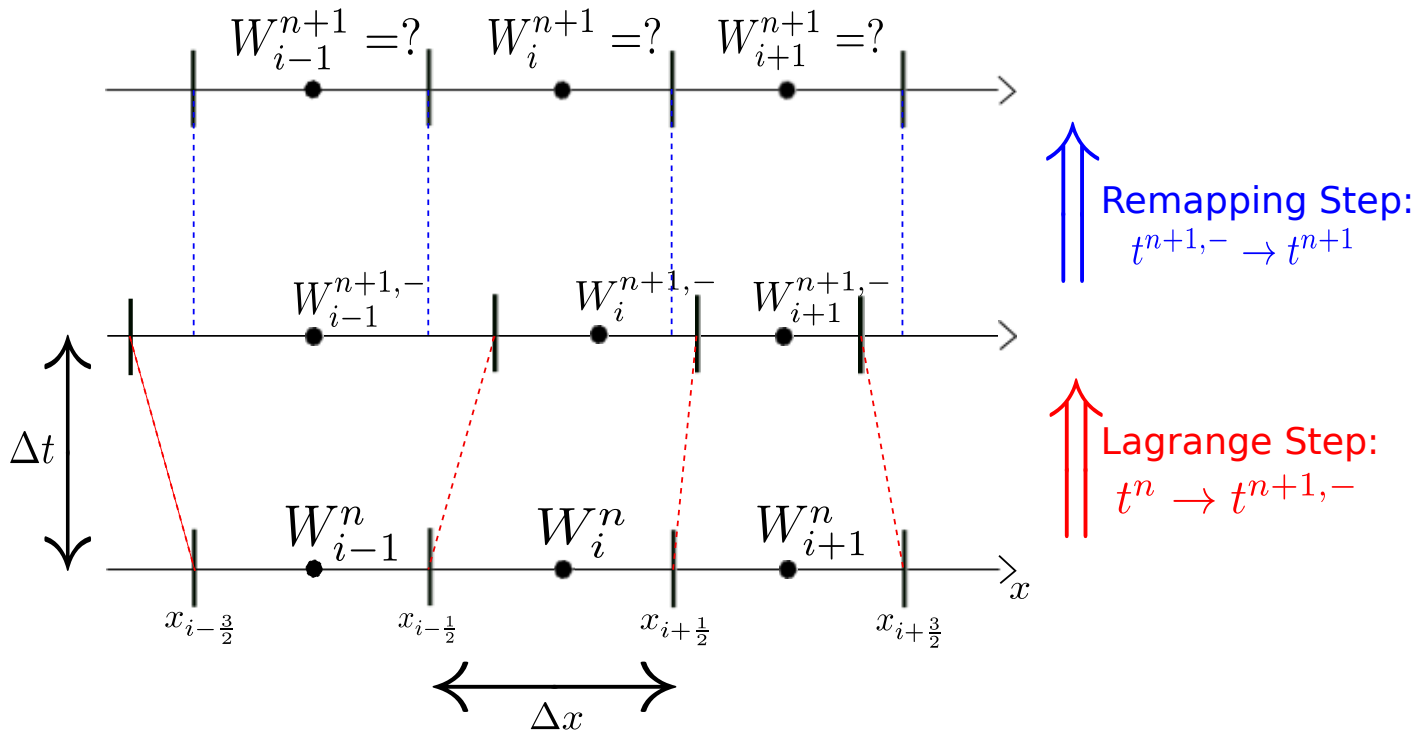
$$u_{i+\frac{1}{2}}^n = \frac{c_{i+1/2,L}^n u_i^n + c_{i+1/2,R}^n u_{i+1}^n + p_i^n - p_{i+1}^n}{c_{i+1/2,L}^n + c_{i+1/2,R}^n},$$

$$p_{i+\frac{1}{2}}^n = \frac{c_{i+1/2,R}^n p_i^n + c_{i+1/2,L}^n p_{i+1}^n + c_{i+1/2,L}^n c_{i+1/2,R}^n (u_i^n - u_{i+1}^n)}{c_{i+1/2,L}^n + c_{i+1/2,R}^n},$$

where $c_{i+1/2,L}^n$ and $c_{i+1/2,R}^n$ are chosen such that our solver satisfies positivity properties.

Remapping step

- The Lagrangian step is done : we have $W_i^{n+1,-}$.
- Problem : how to do the projection to go back to the original grid ?



Solution : a mixed projection

We apply random sampling at the two-fluid interface

- If $(\varphi_{i-1}^n - \frac{1}{2})(\varphi_i^n - \frac{1}{2}) < 0$ or $(\varphi_i^n - \frac{1}{2})(\varphi_{i+1}^n - \frac{1}{2}) < 0$,

  ω_n is a random number $\in [0, 1[$, and we take :

$$W_i^{n+1} = \begin{cases} W_{i-1}^{n+1,-}, \ if \ x_{i-\frac{1}{2}} + \omega_n \Delta x < x_{i-\frac{1}{2}}^{n+1,-}, \\ W_i^{n+1,-}, \ if \ x_{i-\frac{1}{2}}^{n+1,-} \leq x_{i-\frac{1}{2}} + \omega_n \Delta x \leq x_{i+\frac{1}{2}}^{n+1,-}, \\ W_{i+1}^{n+1,-}, \ if \ x_{i-\frac{1}{2}} + \omega_n \Delta x > x_{i+\frac{1}{2}}^{n+1,-}, \end{cases}$$

- else, we use the natural cell averaging projection

$$W_i^{n+1} = W_i^{n+1,-} - \frac{\Delta t}{\Delta x} \quad (\max(u_{i-\frac{1}{2}}, 0)(W_i^{n+1,-} - W_{i-1}^{n+1,-}) + \min(u_{i+\frac{1}{2}}, 0)(W_{i+1}^{n+1,-} - W_i^{n+1,-})).$$

Choice for ω_n

A good choice for the random number ω_n is the (k_1, k_2) Van Der Coput sequence. A implementation in C is given by :

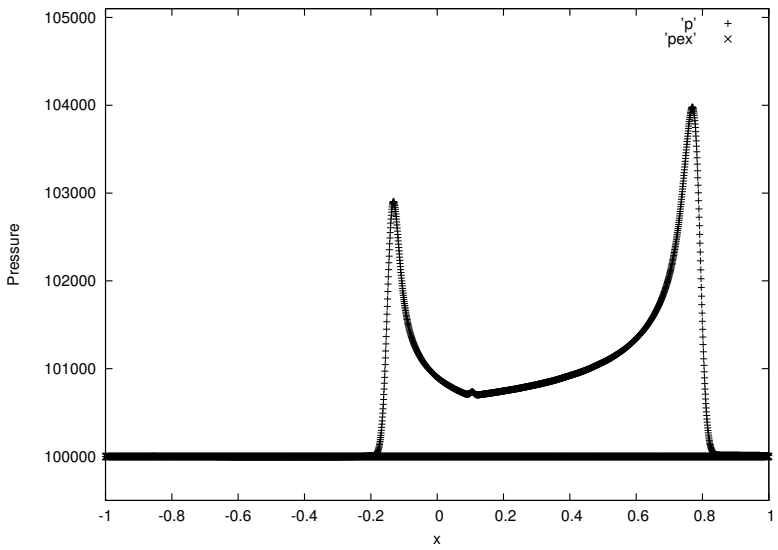
```
float corput ( int n, int k1, int k2){
    float corput = 0;
    float s = 1;
    while(n > 0){
        s/ = k1;
        corput+ = (k2 * n%k1)%k1 * s;
        n/ = k1;
    }
    return corput;
}
```

where k_1 and k_2 are two prime numbers satisfying $k_1 > k_2 > 0$. Practically, we use the $(5, 3)$ Van der Corput sequence. With 200 values, its arithmetic mean is 0.49598 and its standard deviation is 0.28907.

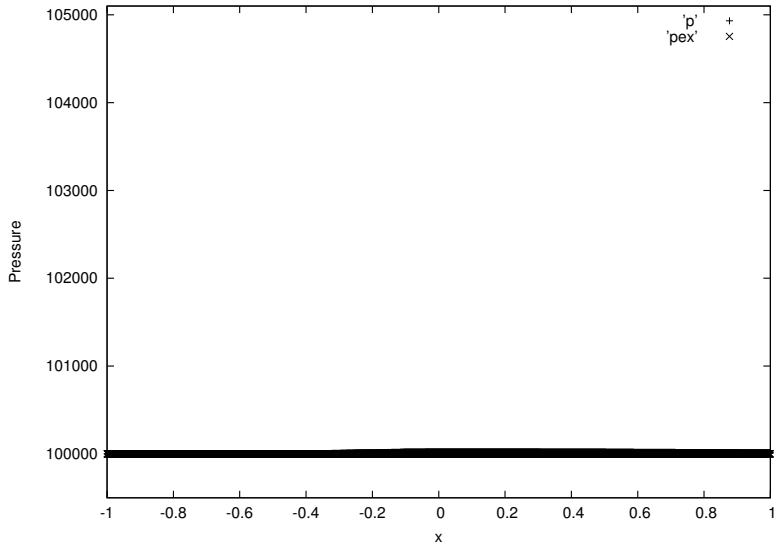
Results obtained with the mixed projection

There is no oscillations on pressure where classical conservative scheme failed.

Classical conservative scheme :



The random scheme gives :



Properties of the random scheme

The random scheme has the following properties :

- it preserves the non negativity of ρ and e,
- it does not diffuse the gas mass fraction φ,
- if at initial time the x-velocity u and the pressure p are constant, this property is preserved at any time.
- it handles vaccum.

Remark : the random scheme is not conservative, even for the total mass.

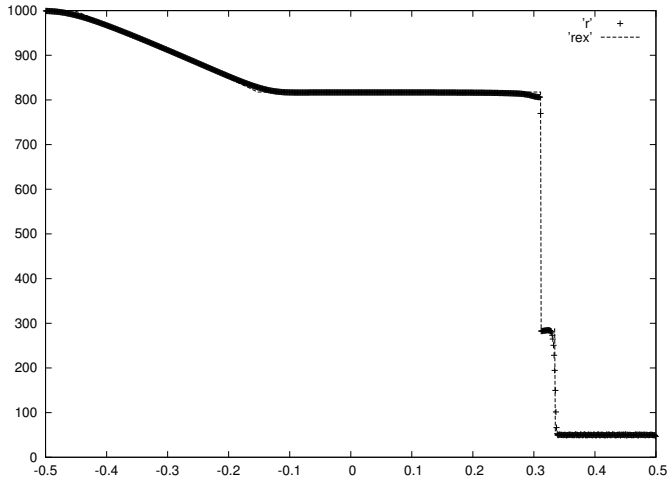
A 1D test

We test the random scheme on a water-air shock tube with discontinuous normal velocity.

Initial condition : We consider

Quantitiés	x<0.2	x>0.2
$\rho(kg.m^{-3})$	1000	50
$u(m.s^{-1})$	0	0
$v(m.s^{-1})$	1000	-5000
$p(Pa)$	1e9	1e5
φ	0	1
γ	4.4	1.4
$\pi(Pa)$	6.8e8	0

We plot the numerical and the exact solution at time $t = 240\mu m$:



A 1D test : comparison with other schemes

We compare the random scheme with other schemes :

- the "ghost fluid for the poor" algorithm of Abgrall [AK2001] (referenced by GF),
- the algorithm of Saurel-Abgrall [SA1999](referenced by SA),
- another scheme (BHRJ) that includes two steps :
  - a lagrangian step : as the random scheme,
  - a projection step : we use a conservative projection on for ρ, ρu, ρv and ρE and for φ, we take :

$$\varphi_i^{n+1} = \varphi_i^{n+1,-} - \frac{\Delta t}{\Delta x} \quad (\max(u_{i-\frac{1}{2}}, 0)(\varphi_i^{n+1,-} - \varphi_{i-1}^{n+1,-}) + \min(u_{i+\frac{1}{2}}, 0)(\varphi_{i+1}^{n+1,-} - \varphi_i^{n+1,-})).$$

These three schemes preserves the total mass.

A 1D test : comparison with other schemes

If we want that the three schemes GF, SA and BHRJ preserve constant (u, p) states, they do not preserve the total energy. Assume that $\forall i,\ (u,p)_i^n = (u,p)$, a sufficient condition to have $(u,p)_i^{n+1} = (u,p)$ is :

$$\begin{aligned}(\rho e)_i^{n+1} &= (\rho e)_i^n - \frac{\Delta t}{\Delta x}(((\rho e + p)u)_{i+1/2} - ((\rho e + p)u)_{i-1/2}),\\ (\rho v^2)_i^{n+1,*} &= (\rho v^2)_i^n - \frac{\Delta t}{\Delta x}(((\rho v^2)u)_{i+1/2} - ((\rho v^2)u)_{i-1/2}),\end{aligned}$$

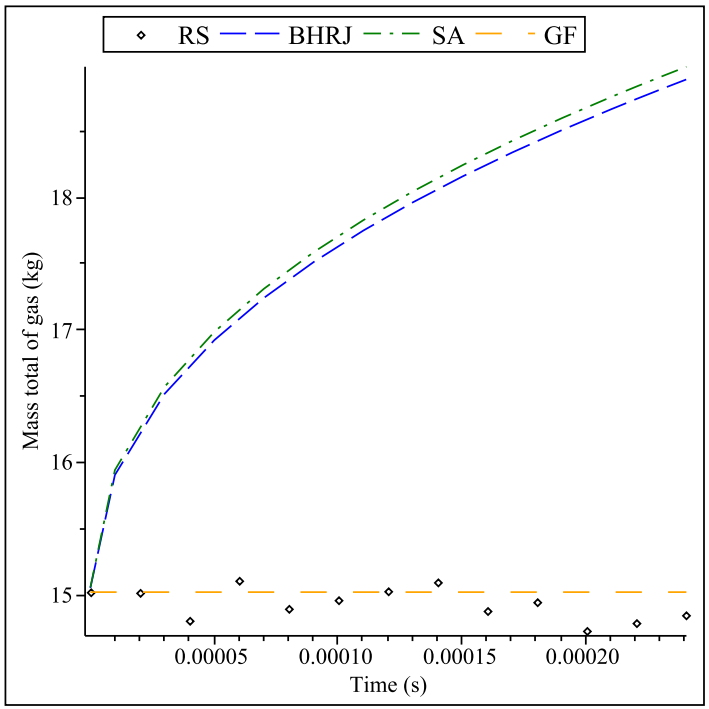
then we perform :

$$(\rho E)_i^{n+1} = (\rho E)_i^{n+1} - (\rho v^2/2)_i^{n+1,*} + (\rho v^2/2)_i^{n+1}.$$

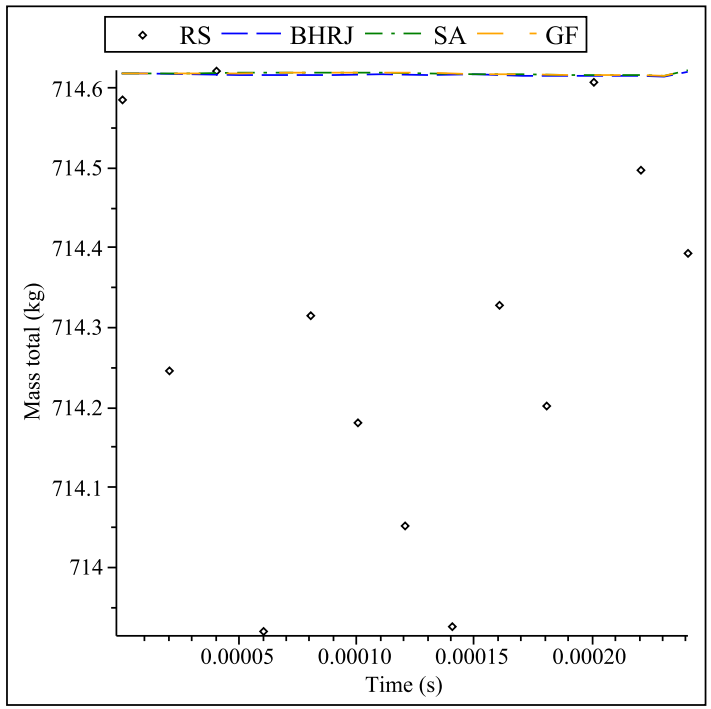
A 1D test : comparison with other schemes

We plot the total mass of gas and the total mass during time $t = 240\mu m$ with approximatively 1000 cells.

Total mass of gas :



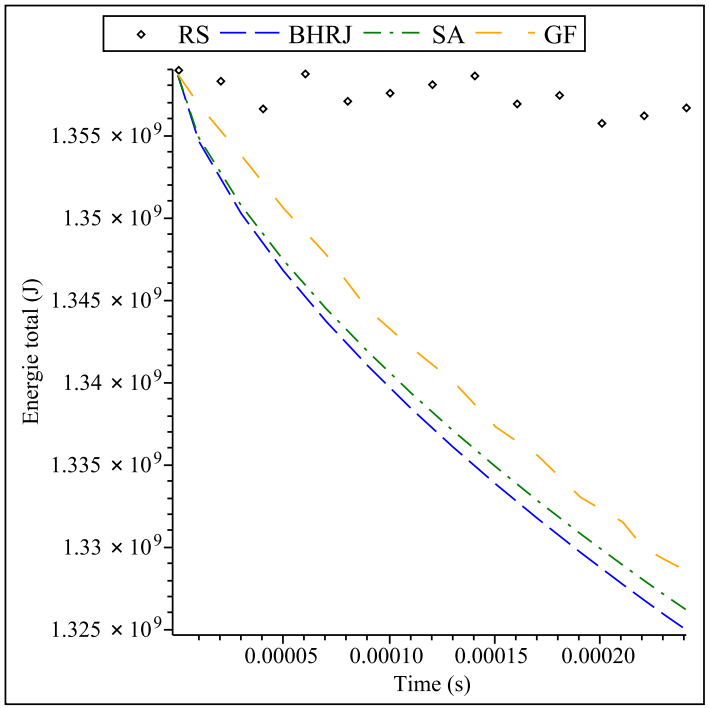
Total mass :



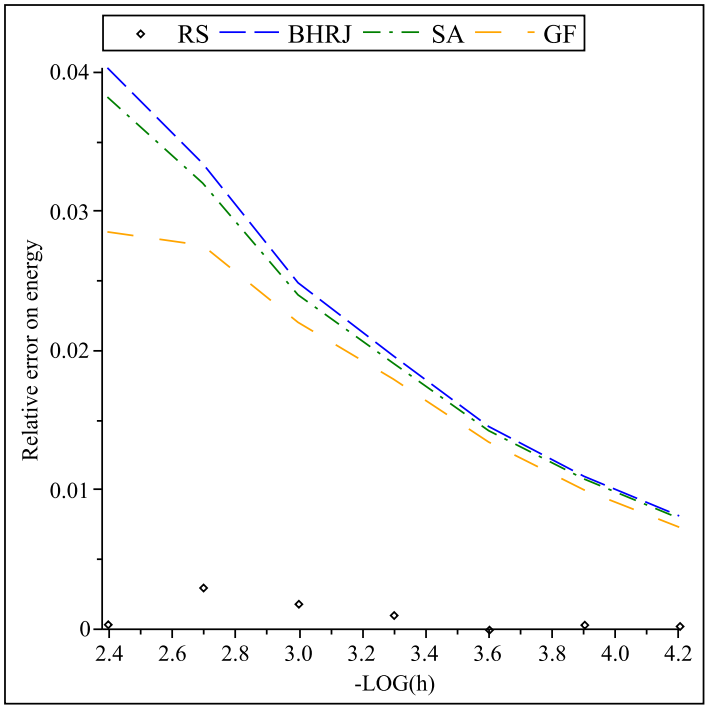
$\Rightarrow$ Error less than 0.1 per cent.

A 1D test : comparison with other schemes

We plot the total energy until time $t = 240\mu m$:



To see if the total energy converge to the exact solution, we plot the relative error on the L^1-norm on energy :



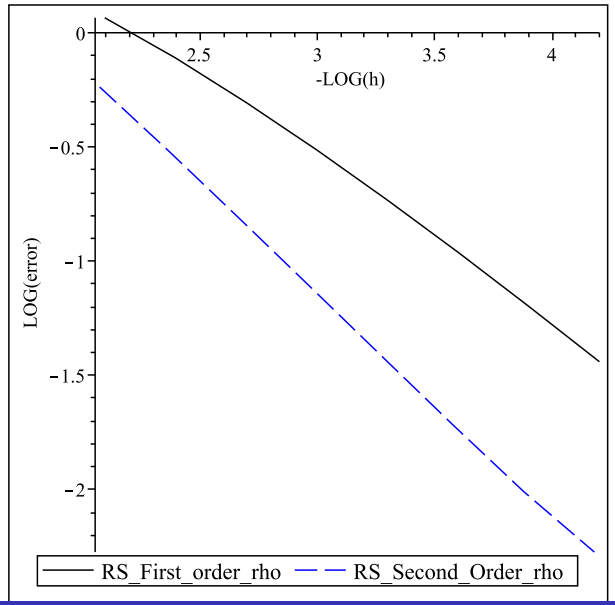
Second order

- We pass to the second order in the lagrangian step with a MUSCL reconstruction on the variable (ρ, u, v, p, φ). We couple this reconstruction with a Heun scheme.

The second order scheme has the same properties :

- it preserves the non-negativity of ρ and e,
- it handles vacuum,
- it does not diffuse the gas mass fraction φ,

We plot the convergence curve at first and second order, we see the improvement :



Implementation on GPU : speedup

The computation corresponds to 300 iterations on a grid 1024×512.

	time (s)
AMD A8 3850 (1 coeur)	527
AMD A8 3850 (4 coeurs)	205
NVIDIA GeForce 320M	56
AMD Radeon HD5850	3
AMD Radeon HD7970	2

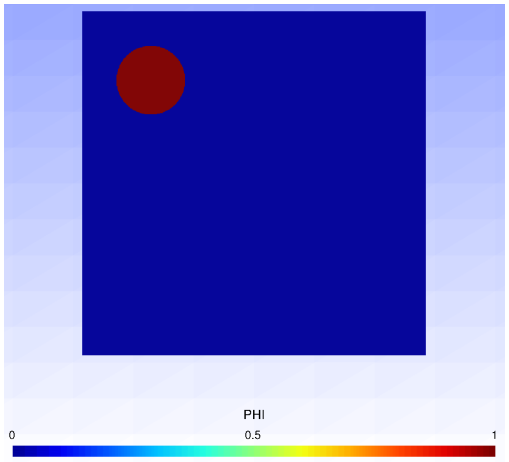
Advection of a bubble of gas in uniform liquid flow

We test the random sampling coupling with the dimensional splitting on a pure advection flow.

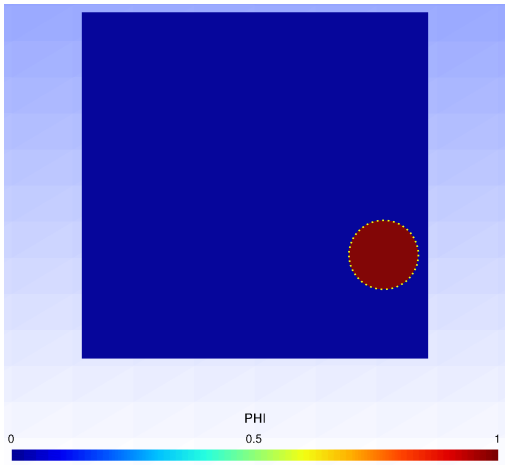
- Initial datas :

Quantities	Bubble	Out.
$\rho(kg.m^{-3})$	1.225	1000
$u(m.s^{-1})$	100	100
$v(m.s^{-1})$	-75	-75
$p(Pa)$	1.01e5	1.01e5
φ	1	0
γ	1.4	3
π	0	7.5e8

- Initial time :



- At $6.7\,10^{-3}s$:



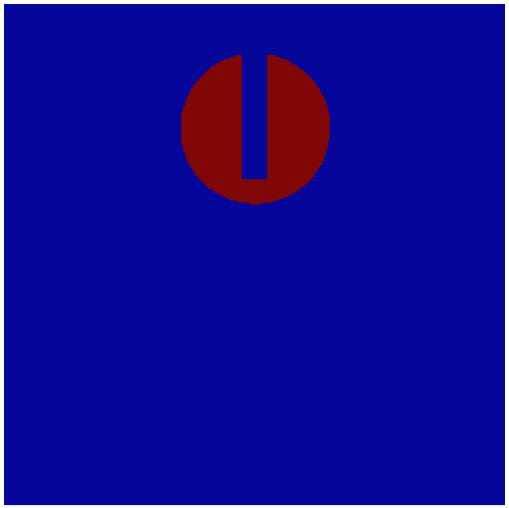
Test of Zalesak [Z1979]

We consider a solid body rotation. We choose

$$u = -\Omega(y - y_0), \qquad v = \Omega(x - x_0),$$

where $\Omega = \frac{2\pi}{628}$ is the angular velocity and (x_0, y_0) is the axis of rotation.

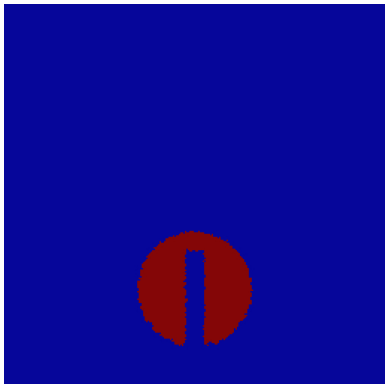
- Initial time :



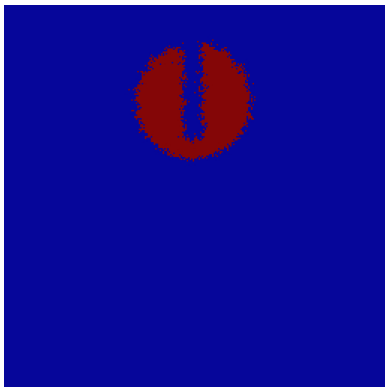
Test of Zalesak [Z1979]

Results with RS algorithm :

- $\frac{1}{2}$ revolution :

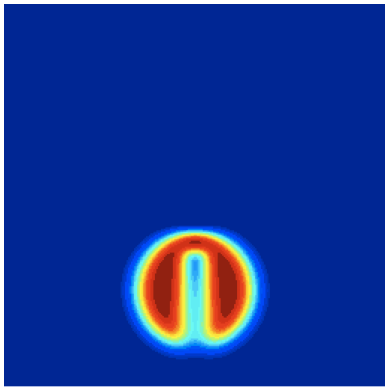


- 5 revolutions :

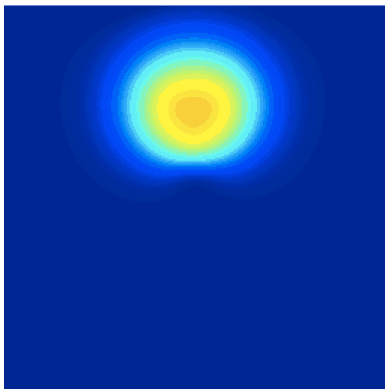


Results with BHRJ algorithm :

- $\frac{1}{2}$ revolution :

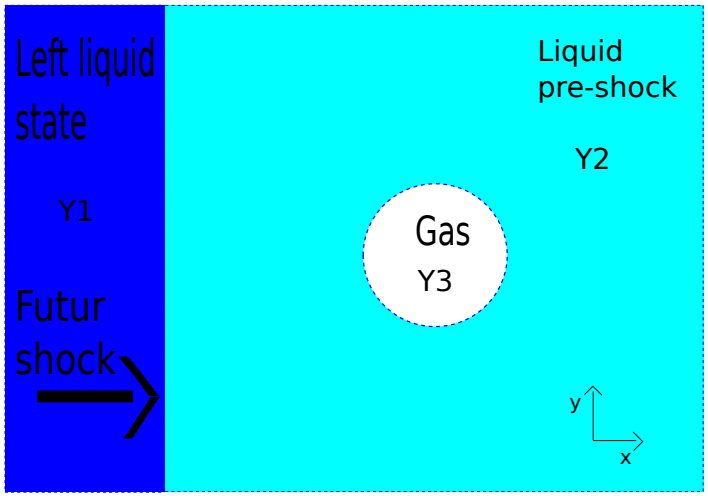


- 5 revolutions :



Shock bubble interaction (water/gas)

We consider the shock-bubble interaction test of [KL2010]



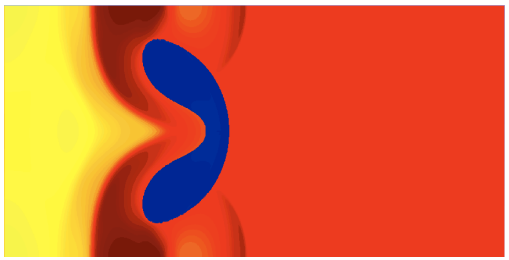
The initial data are :

Quantités	Y1	Y2	Y3
$\rho(kg.m^{-3})$	1030.9	1000	1
$u(m.s^{-1})$	300	0	0
$v(m.s^{-1})$	0	0	0
$p(Pa)$	3.0e9	1.0e5	1.0e5
φ	0	0	1
γ	4.4	4.4	1.4
π	6.8e8	6.8e8	0

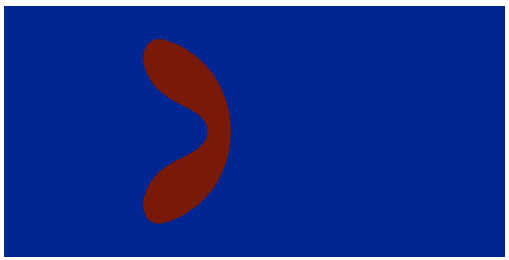
Final time=$600\mu s$.
Grid : 3000×1500.
Time of computation on GPU (Tahiti) : $360s \approx 6min$.

Results with RS :

- Density at $375\mu s$:

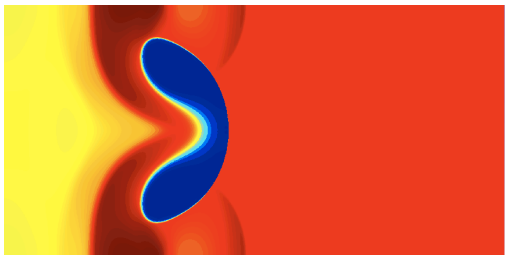


- φ at $375\mu s$:

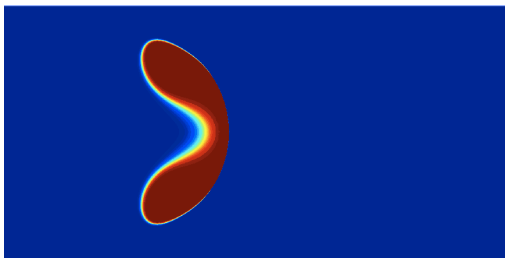


Results with BHRJ :

- Density at $375\mu s$:

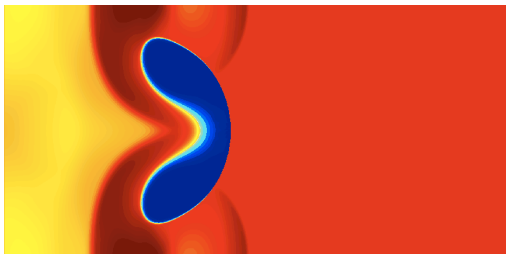


- φ at $375\mu s$:

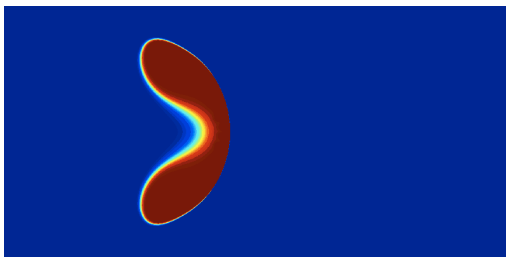


Results with SA :

- Density at $375\mu s$:

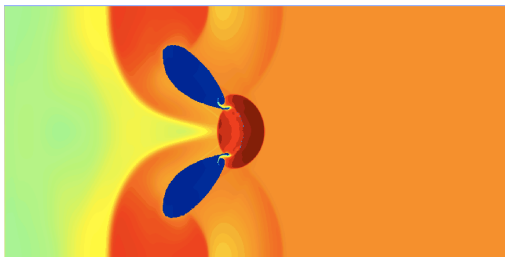


- φ at $375\mu s$:

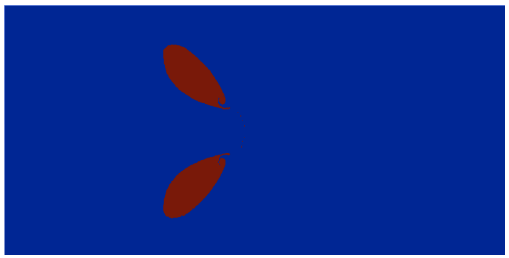


Results with RS :

- Density at $450\mu s$:

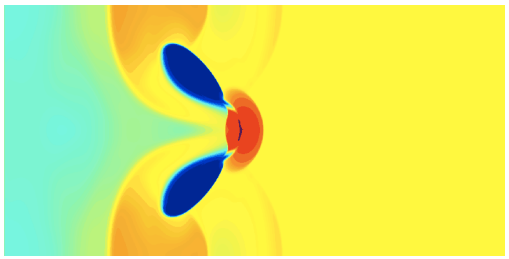


- φ at $450\mu s$:

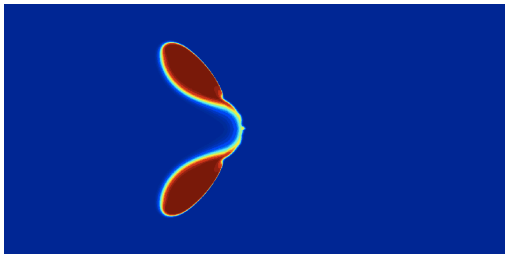


Results with BHRJ :

- Density at $450\mu s$:

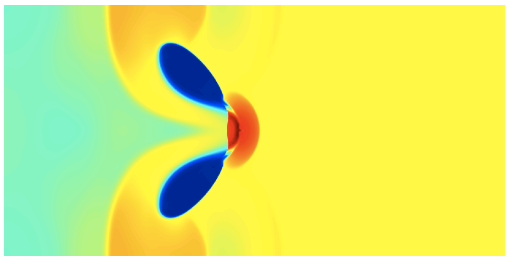


- φ at $450\mu s$:

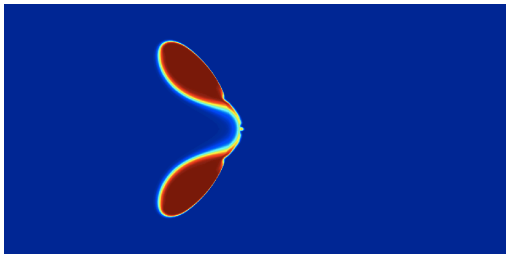


Results with SA :

- Density at $450\mu s$:

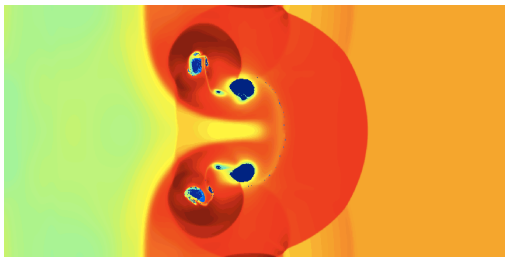


- φ at $450\mu s$:

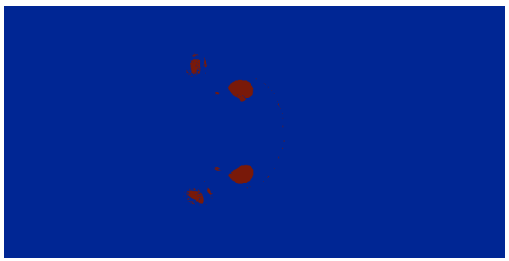


Results with RS :

- Density at $600\mu s$:

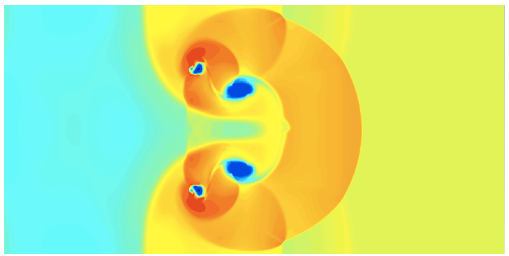


- φ at $600\mu s$:

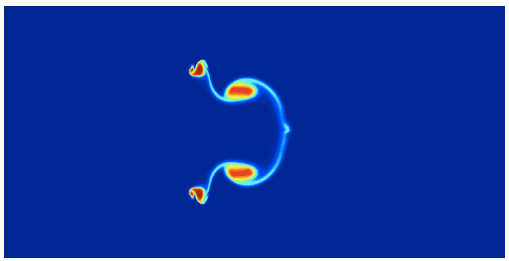


Results with BHRJ :

- Density at $600\mu s$:



- φ at $600\mu s$:



Results with SA :

- Density at $600\mu s$:

  Problem with the vpn.

- φ at $600\mu s$:

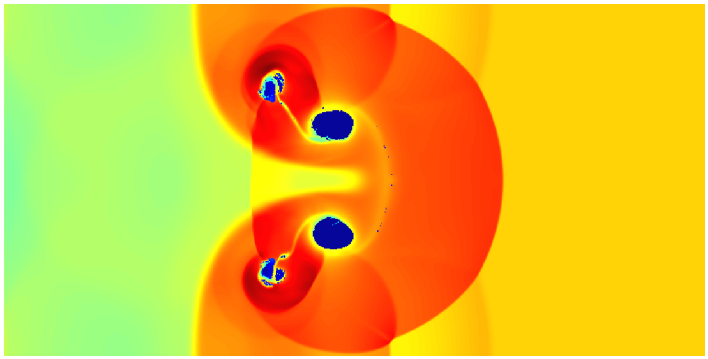
  Problem with the vpn

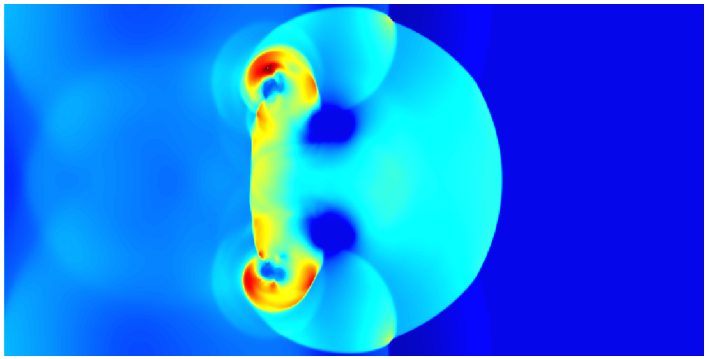
Conclusion

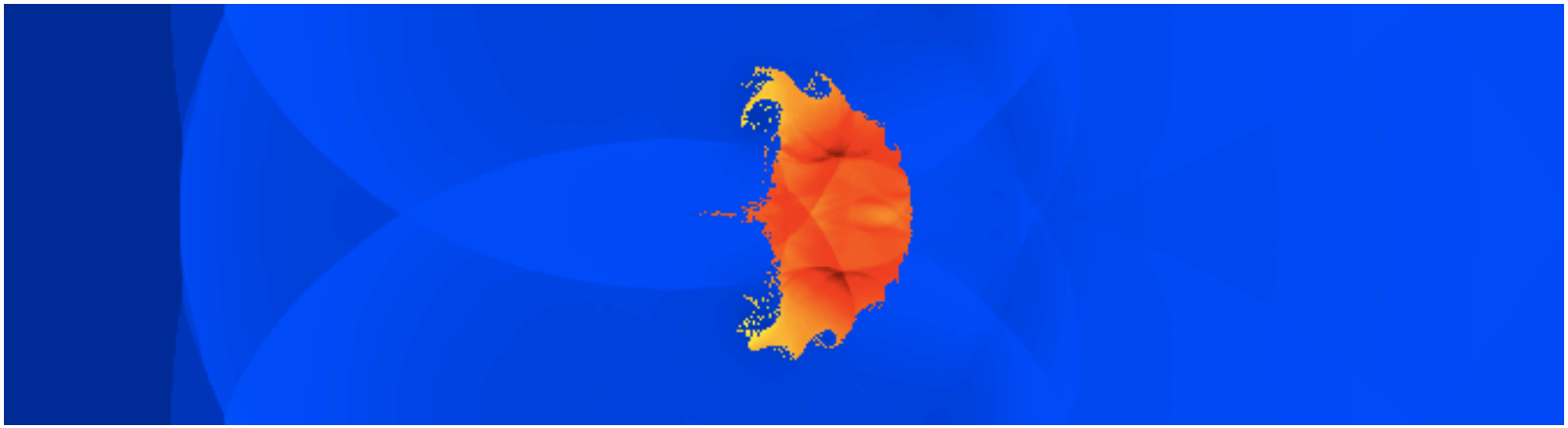
- We developed an efficient algorithm to treat complex compressible flows.
- The RS has the following properties :
  - it preserves the non-negativity of ρ and e,
  - it does not diffuse the gas mass fraction φ.
- The code is very efficient on GPU, we need few minutes to compute a complex flow on a mesh with millions of cells.

- Futur works :
  - Test the MPI version of the code (multi GPU).
  - Consider axysymetrical geometry (bubble of cavitation).

Thank you for your attention !







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