3. Intégration numérique

3.4 Exercices

Exercice 17

```
> retart :
```

```
> intrecd := proc( f, a, b, N )
  local h :
  h := (b − a) / N :
  h·add( subs( x = a + k·h, f ), k = 1 ..N ) :
  end proc:
```

```
> intrecd(1, 0, 2, 10);
```

$$2 \qquad \textbf{(1.1.1.1)}$$

```
> evalf(intrecd(x, 0, 1, 10000));
```

$$0.5000500000 \qquad \textbf{(1.1.1.2)}$$

Exercice 15

```
> inttrap := proc( f, a, b, N )
  local h, s :
  h := (b − a) / N :
  s := h/2 · ( subs( x = a, f ) + subs( x = b, f ) ) +
  h·add( subs( x = a + k·h, f ), k = 1 ..N − 1 ) :
  s :
  end proc:
```

```
> inttrap(1, 0, 2, 10);
```

$$2 \qquad \textbf{(1.1.2.1)}$$

```
> inttrap(x, 0, 1, 10);
```

$$\frac{1}{2} \qquad \textbf{(1.1.2.2)}$$

```
> evalf( inttrap( x^2, 0, 1, 1000 ) );
```

$$0.3333335000 \qquad \textbf{(1.1.2.3)}$$

```
> evalf( 1/3 );
```

$$0.3333333333 \qquad \textbf{(1.1.2.4)}$$

```
>
```

Exercice 16

```
> intsimp := proc(f, a, b, N)
  local h:
  h := (b - a)/N:
  add((h/3)·subs(x = a + 2·k·h, f) +
  (4·h/3)·subs(x = a + (2·k + 1)·h, f) +
  (h/3)·subs(x = a + (2·k + 2)·h, f), k = 0 .. N/2 - 1):
  end proc:
```

```
> intsimp(1, 0, 2, 10);
```

$$2 \qquad \textbf{(1.1.3.1)}$$

```
> intsimp(x, 0, 1, 10);
```

$$\frac{1}{2} \qquad \textbf{(1.1.3.2)}$$

```
> intsimp(x^2, 0, 1, 10);
```

$$\frac{1}{3} \qquad \textbf{(1.1.3.3)}$$

```
> intsimp(x^3, 0, 1, 10);
```

$$\frac{1}{4} \qquad \textbf{(1.1.3.4)}$$

```
> evalf(intsimp(x^4, 0, 1, 100));
```

$$0.2000000013 \qquad \textbf{(1.1.3.5)}$$

```
> evalf(1/5);
```

$$0.2000000000 \qquad \textbf{(1.1.3.6)}$$

```
>
```

Exercice 20

```
> V := [0, 19.5, 35, 45, 40.5, 25, 20.5, 29, 27, 12.5, 0];
  N := nops(V) - 1;
```

$$V := [0, 19.5, 35, 45, 40.5, 25, 20.5, 29, 27, 12.5, 0]$$

$$N := 10 \qquad \textbf{(1.1.4.1)}$$

```
> f := x→x^2:
  V2 := map(f, V);
```

$$V2 := [0, 380.25, 1225, 2025, 1640.25, 625, 420.25, 841, 729, 156.25, 0] \qquad \textbf{(1.1.4.2)}$$

```
> a := 0 : b := 0.1 : h := (b - a)/N :
> s := add( h/3 · V2[2·k + 1] + 4·h/3 · V2[2·k + 1 + 1] +
  h/3 · V2[2·k + 2 + 1], k = 0 .. N/2 - 1 );
```

$$s := 80.46333331 \qquad \textbf{(1.1.4.3)}$$

```
> Vrms := sqrt( 1/(b - a) · s );
```

$$Vrms := 28.36605953 \qquad \textbf{(1.1.4.4)}$$

```
>
```

5. Equations différentielles

5.4 Implémentation

```
> #Méthode d'Euler explicite
> euler2 := proc( f, y0, t0, T, N )
  local h, y, n;
  h := (T - t0)/N :
  y := array(0 .. N ) :
  y[0] := y0 :
  for n from 0 to N - 1 do
  y[n + 1] := y[n] + h·f (t0 + n·h, y[n]) :
  od:
  end proc:
>
> #Methode de Runge
> runge := proc( f, y0, t0, T, N )
  local h, y, n, k1, k2;
  h := (T - t0)/N :
  y := array(0 .. N ) :
  y[0] := y0 :
  for n from 0 to N - 1 do
  k1 := f (t0 + n·h, y[n]) :
  k2 := f( t0 + n·h + h/2, y[n] + h/2 · k1 ) :
  y[n + 1] := y[n] + h·k2 :
  od:
  end proc:
```

```
>
> #Methode de Runge-Kutta d'ordre 4
> runge := proc( f, y0, t0, T, N )
  local h, y, n, k1, k2, k3, k4;
  h := (T - t0) / N :
  y := array(0..N) :
  y[0] := y0 :
  for n from 0 to N - 1 do
  k1 := f(t0 + n·h, y[n]) :
  k2 := f(t0 + n·h + h/2, y[n] + h/2·k1) :
  k3 := f(t0 + n·h + h/2, y[n] + h/2·k2) :
  k4 := f(t0 + (n + 1)·h, y[n] + h·k3) :
  y[n + 1] := y[n] + h·(k1 + 2·k2 + 2·k3 + k4) / 6 :
  od:
  end proc:
>
```