Quantitative Risk Analysis
Master of Financial Engineering - M2
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Third practical : Importance sampling - Variance Reduction

In this practical, we want test the importance sampling method. Assume that we want estimate

$$\theta = P(X \geq 3)$$

where X follows a standard normale law.

1) Using Box-Miller random number generator, approximate θ with a Monte-Carlo method with $N = 1000$ realizations. What is the problem ?
2) Implement error control in the function written in the previous question. This new function will take as arguments a probability p (confidence level for error control) and a value ϵ (maximal absolute error for error control). Error will be implemented using the sequential method.
3) Estimate θ with $p = 0.99$ and $\epsilon = 0.0005$.
4) Write θ as the integral of a function, using the importance sampling method and the density $g(x)$ of a normal law with average $\mu = 3$ and standard deviation $\sigma = 1$.
5) Write a Matlab function that estimates θ by carrying out Monte-Carlo simulations. Random variable generation will be done using the importance sampling method with the formula obtained in the question 4). Implement error control in the previous function.
6) Using the importance sampling, estimate θ with $p = 0.99$ and $\epsilon = 0.0005$. Tray with $\epsilon = 0.0003$.

Assume now that we want estimate

$$\theta = E_f(h(X))$$

where X follows a standard normal law and for all $x \in \mathbb{R}$

$$h(x) = x^4 \exp\left(\frac{X^2}{4}\right) \mathbf{1}_{[2,+\infty[}(x).$$

7) Approximate θ with a Monte-Carlo method.
8) Can approximate θ as well as you want.
9) Write θ as the integral of a function, using the importance sampling method and the density $g(x)$ of a normal law with average μ and standard deviation $\sigma = 1$.
10) Choose μ such that the maximum principle is satisfied.
11) Using the importance sampling, estimate θ with $p = 0.99$ and $\epsilon = 0.0005$.
12) Choose an other μ, what is the consequence on the precision of the estimation ?