Quantitative Risk Analysis
Master of Financial Engineering - M2
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Second Practical : Conditional Value-at-Risk

The CVaR is a risk measure that solves some shortcomings of the VaR. This measure is the average loss *when the loss is greater than the VaR*. In particular, this is a conditional expectancy :

$$\mathrm{CVaR}_\alpha(X) = \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_u(X)du = \frac{1}{1-\alpha}\int_{\mathrm{VaR}_\alpha(X)}^{+\infty} xf(x)dx,$$

where $x \mapsto f(x)$ is the density function of random variable X.

Assume that the loss X at time $T > 0$ of an investment follows a uniform distribution between -30 and 10.

1) Compute the exact value of $CVaR_{0.9}(X)$.
2) Write a Matlab function $CVaR$ that takes as arguments a table $[x_1, \cdots, x_n]$ of realizations of a random variable X and a confidence level $\alpha \in]0,1[$ and returns $VaR_\alpha(X)$ obtained with a Monte-Carlo method.
3) Use the function built in question 2) to approximate $CVaR_{0.9}(X)$ with a Monte-Carlo method with $n = 10\,000$ realizations.

Consider the two securities $i = 1$ et $i = 2$ described in the previous practical whose respective loss at time T are random variables X_1 and X_2.

4) Write a Matlab function that estimates $\mathrm{CVaR}_\alpha(X_1+X_2)$ using Monte-Carlo simulations. This function will take as arguments confidence level α and a number n of simulations to perform.
5) Estimate $\mathrm{CVaR}_\alpha(X_1 + X_2)$ with $\alpha = 0.99$ and $10\,000$ simulations.
6) Implement error control in the function written in previous question. This new function will take as arguments a probability p (confidence level for error control), a value ε (maximal relative error for error control), and confidence

level α for the calculation of the CVaR. Error control will be implemented using the sequential method.

7) Estimate $\text{CVaR}_\alpha(X_1 + X_2)$ with $\alpha = p = 0.99$ and $\varepsilon = 0.05$.

Now consider a security whose loss at time T is given by random variable X of density :

$$f(x) = \begin{cases} -\frac{0.96}{288}(x-5) & \text{si } x \in [-19; 5], \\ 1 & \text{si } x \in [20; 20.04], \\ 0 & \text{sinon.} \end{cases}$$

8) Depict function $f(x)$ and compute $\int_{-\infty}^{+\infty} f(x)dx$.
9) Compute the exact values of $\text{VaR}_{0.95}(X)$ and $\text{CVaR}_{0.95}(X)$.
10) Write $\text{CVaR}_\alpha(X)$ as the integral of a function, using the importance sampling method and the density $g(x)$ of a normal law with average μ and standard deviation σ.
11) Write a Matlab function that estimates the value of $\text{CVaR}_\alpha(X)$ by carrying out Monte-Carlo simulations. Random variable generation will be done using the importance sampling method with the formula obtained in question 10). This function will take as arguments confidence level α, the number n of simulations to carry out, the average μ and the standard deviation σ of density g.
12) Implement error control in the function written in previous question. This new function will take as arguments a probability p (confidence level for error control) and a value ε (maximal relative error for error control) in addition to the parameters already used. Error control will be implemented using the sequential method.
13) What values of μ give the most precise evaluation of $\text{CVaR}_\alpha(X)$ in the implementation done in question 12) ?