Quantitative Risk Analysis
Master of Financial Engineering - M2
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First Practical : Value-at-Risk

Consider a portfolio of value V_0 at time $t = 0$ and V_T at time $T > 0$. We call *incurred loss at time* T the following random variable :

$$X = V_0 - V_T.$$

The value-at-Risk of this portfolio at confidence level α reads :

$$VaR_\alpha(X) = \min\left\{g : P(X \leq g) \geq \alpha\right\}.$$

According to this definition, the probability that the incurred loss at time T be greater than VaR_α is $1 - \alpha$. In other words, VaR_α is the maximal loss of this portfolio at confidence level α.

1) It is assumed in this question that X is normally distributed with an average of 0 and a standard deviation of 1. What is $\mathrm{VaR}_{0.9}(X)$?
2) Write a Matlab function VaR that takes as arguments a table $[x_1, \cdots, x_n]$ of realizations of a random variable X and a confidence level $\alpha \in]0, 1[$ and returns $VaR_\alpha(X)$ obtained with a Monte-Carlo method.
3) Use your Matlab function VaR to compute $VaR_{0.9}(X)$ where X is a standard normal random variable. The realizations of the normal random variables will be built using Box and Muller method.

Now consider two independent securities $i = 1$ and $i = 2$. It is assumed that the loss at time $T > 0$ of security i is modeled as random variable X_i. Variable X_i is almost normally distributed. More precisely :

$$X_i = \beta_i + \eta_i,$$

where β_i follows a standard normal law and where the distribution of η_i is :

$$P(\eta_i = 0) = 0.991 \text{ and } P(\eta_i = 10) = 0.009.$$

4) Using the definition of the *Value-at-Risk*, give the equation satisfied by $\mathrm{VaR}_\alpha(X_1 + X_2)$. How can you solve this equation for $\mathrm{VaR}_\alpha(X_1 + X_2)$?
5) Using random number generators developed in the course of Advanced Algorithms, write a Matlab function that generates a vector of n realizations of random variable X_i.
6) Using the function defined in the course of Advanced Algorithms for finance, plot the density and the cumulative of the variable X_1 with $N = 10000$ realizations.
7) Compute $\mathrm{VaR}_{0.99}(X_1)$ and $\mathrm{VaR}_{0.99}(X_2)$ using Monte-Carlo simulations with $n = 10\,000$ realizations.
8) Write a Matlab function that estimates $\mathrm{VaR}_\alpha(X_1 + X_2)$ using Monte-Carlo simulations. This function will take as arguments the confidence level α and a number n of simulations to perform.
9) Use your previous function to compute $VaR_{0.9}(X_1 + X_2)$ and $VaR_{0.99}(X_1 + X_2)$ with $n = 10\,000$ simulations.
10) Implement error control in the function written for question 8). Your new function should take as arguments a probability p (confidence level for the error control), a value ε (maximal relative error), and confidence level α for the computation of the VaR. Error control will be done using the sequential method.
11) Estimate $\mathrm{VaR}_\alpha(X_1 + X_2)$ with $\alpha = p = 0.99$ and $\varepsilon = 0.05$.