Quantitative Risk Analysis
Master of Financial Engineering - M2
Final exam - Duration : 2 hours
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No documents - You can use a calculator

Please report clear and detailed answers explicitly, as well as all the formulas you use and all the theorems you invoke.

Problem 1 :

Consider a security whose **prices** y_0, ..., y_{10} during the last 11 days have been :

$$\begin{array}{llllll} y_0 = 10, & y_1 = 8, & y_2 = 5, & y_3 = 12, & y_4 = 4, & y_5 = 2, \\ y_6 = 1, & y_7 = 5, & y_8 = 1, & y_9 = 3, & y_{10} = 2. & \end{array}$$

The daily loss is the loss on the price of the security between a given day and the following day.

1) Compute the 10 daily looses $x_1, ..., x_{10}$ that correspond to the 11 prices of the security.
2) Compute the historic VaR of these yields with $\alpha = 0.90$.
3) Compute the historic CVaR of these yields with $\alpha = 0.90$.

Problem 2 :

Consider a security whose price at time $t = 0$ is 100 euros and whose **price** at time $T > 0$ is a random variable X exponentially distributed with an average of 105 euros.

We recall that the density of the exponential distribution with average μ is

$$f(x) = \begin{cases} \frac{1}{\mu} e^{-\frac{x}{\mu}}, & \text{if } x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

1) Compute the 0.95-VaR of the loss at time T.
2) Compute the 0.95-CVaR of the loss at time T.

Problem 3 :

Assume we need to estimate

$$\theta = P(X \geq 3)$$

where X follows a standard normal law.

We recall that the density of the normal distribution with average μ and standard deviation is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma}\right)$$

We will use a Monte-Carlo method to estimate θ.

1) Write θ as

$$\theta = E_f(h(X))$$

for some function $h : \mathbb{R} \to \mathbb{R}$.

2) Assume that

$$x_1 = -1, \;\; x_2 = 0, \;\; x_3 = 1$$

are realizations following density f. Give an estimation θ_3 of θ by Monte-Carlo using these three realizations.

3) Give a confidence interval at level $p = 95\%$ for θ. What is the problem ?

4) Using the importance sampling method and the density $g(x)$ of a normal law with average μ and standard deviation $\sigma = 1$, write θ as

$$\theta = E_g\left(\bar{h}(X)\right)$$

for some function $\bar{h} : \mathbb{R} \to \mathbb{R}$.

5) Describe the maximum principle method and choose μ such that the maximum principle is satisfied.

6) Assume that

$$x_1 = -2, \;\; x_2 = 3, \;\; x_3 = 4$$

are realizations following density g. Give an estimation θ_3 of θ by Monte-Carlo using these three realizations.

7) Give a confidence interval at level $p = 95\%$ for θ based on the importance sampling method.