Advanced Algorithms for Finance
Master of Financial Engineering - M2
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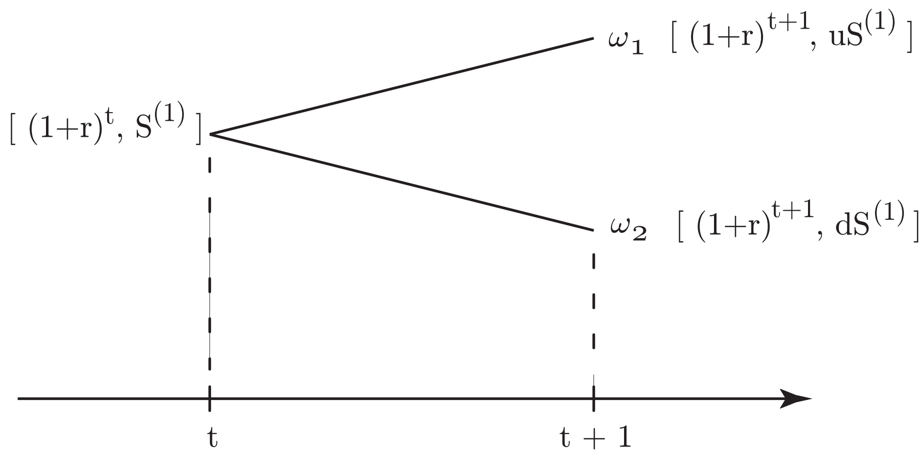
Third Practical : A (naive) Pricer

Thre aim of this practical is to code an option pricer with a simple model for the underlying security. It will be assumed that the market is made up of a numeraire with risk-free rate r, and of a risked security. This is a model over several periods, from time $t = 0$ to time $t = T$. Moreover, at each time t and in each state ω_j, the situation can evolve in exactly two ways from time t to time $t + 1$:

- The value of the risked security increases from $S^{(1)}$ to $uS^{(1)}$, where $u > 1$,
- The value of the risked security decreases from $S^{(1)}$ to $dS^{(1)}$, where $d < 1$.

Note that it is *not* assumed here that d is equal to $1/u$ as in the binomial model. This model can therefore be decomposed into a sequence of models over one period that each can be represented as follows :



1. At time t, how many possible states are there in this model ?
2. Show that there exists a risk-neutral probability distribution in the model over one period shown above if and only if $d < 1 + r < u$.

In the following questions, we search the price at time $t = 0$ of an option whose underlying security is the above risked security. The strike and the expiration date of the option are respectively K and T.

3. First assume that $T = 1$. Using Matlab, write a function that takes arguments r, u, d, K and $S_0^{(1)}$ (the value of the risked security at time $t = 0$) and that returns the value at time $t = 0$ of a european call option.
4. Assuming that the risk-free rate is $r = 0.03$, give the value of a european call of strike 95\$ and expiration date $T = 1$ over an underlying security of price $S = 100$\$ at time $t = 0$, whose variations are modeled with $u = 1.05$ and $d = 0.95$.
5. In this question, T is not necessarily equal to 1. Using question 3), write a recursive function that takes arguments r, u, d, K, t, T, and $S_t^{(1)}$ (the value of the risked security at time $t < T$) and that returns the value at time t of a european call option.
6. In this question, T is not necessarily equal to 1. Write a function that takes arguments r, u, d, K, t, T, and $S_t^{(1)}$ (the value of the risked security at time $t < T$) and that returns the value at time t of a european call option.
7. Now assume that the risk-free rate is $r = 0.02$. Give the value of a european call of strike 95\$ and expiration date $T = 3$ over an underlying security of price $S = 100$\$ at time $t = 0$, whose variations are modeled with $u = 1.05$ and $d = 0.95$.
8. Using the values given in question 6), but with a risk-free rate of $r = 0.04$, what would be the value at $t = 0$ of the same option ? How do you explain the variation of this value ?
9. Write a new function that returns the value of a european put.
10. Assuming that the risk-free rate is $r = 0.02$, give the value of a european put of strike 105\$ and expiration date $T = 3$ over an underlying security of price $S = 100$\$ at time $t = 0$, whose variations are modeled with $u = 1.05$ and $d = 0.95$.
11. Give the value of the put described in the previous question when $T = 10$, $T = 20$, and $T = 30$. What do you observe ?

12. We now consider american options. What is the differentiates these options from european options ? How does it changes the calculations required to price an option ?
13. Write a new recursive function that returns the value of an american call.
14. Assuming that the risk-free rate is $r = 0.006$, give the value of a american call of strike 110$ and expiration date $T = 3$ over an underlying security of price $S = 100\$$ at time $t = 0$, whose variations are modeled with $u = 1.05$ and $d = 0.95$.
15. What is the required modification needed to price an american put ? Implement this modification in a recursive function in order to obtain a pricer for american put options.
16. Now assume that $d = 1/u$. How can the computation procedure you have used be improved in this particular case ? Implement your proposition, and test it with the options described in questions 4), 7),8), 10), and 14) replacing d by $1/u$ and T by 10, 20, and 30. What do you observe ?