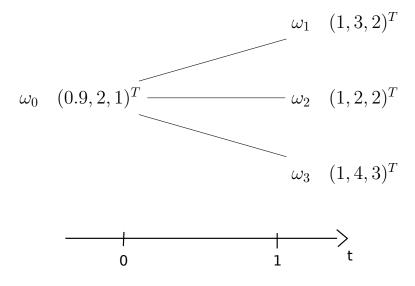
Advanced Algorithms for Finance Master of Financial Engineering - M2 Intermediate Exam - Duration : 1 hour 30 Jung Jonathan



No documents - You can use a calculator

Please report clear and detailed answers explicitly, as well as all the formulas you use and all the theorems you invoke.

Problem 1: We consider the following model over one period, made up of three securities numbered i = 0, 1, and 2:



The vectors given in this figure provide the values of the three securities in every state of the model.

- 1) Is this model binomial? Why?
- 2) Is there a numeraire security? If there exists one, what is the risk-free rate r associated to this numeraire?
- 3) Is a security of price X = (4, 5, 7) at time t = 1 attainable? Do the computation explicitly.
- 4) Is the market complete? Why?
- 5) Prove that the linear pricing hypothesis is satisfied.

Further assume that the linear pricing hypothesis is satisfied.

- 6) What is the value at time t = 0 of a security associated to portfolio $\theta = (2, 1, 3)$ at time t = 1?
- 7) Is there a vector π of state prices? Justify.
- 8) Is there a risk-neutral probability distribution Q?
- 9) Is there an arbitrage portfolio $\theta \in \mathbb{R}^3$?

Problem 2: We consider a binomial model over two periods (T = 2) with u = 1.06, d = 1/u and a risk-free rate r = 0.02.

We assume that the initial value of the numeraire is 1 and that the initial value of the risked security is 100.

- 1) Sketch the tree that corresponds to this model over two periods.
- 2) Show that there exists a risk-neutral probability distribution in the model over one period and give it.
- 3) Give the definition of a call.
- 4) Give the definition of a european call.
- 5) Give the value of a european call of strike K = 105 and expiration date T = 2.