No documents - <u>You can use a calculator</u>

> Please report clear and detailed answers explicitly, as well as all the formulas you use and all the theorems you invoke.

Problem 1 : We consider the following model over one period, made up of three securities numbered $i = 0$, 1, and 2 :

The vectors given in this figure provide the values of the three securities in every state of the model.

1) Is this model binomial ? Why ?
2) Is there a numeraire security ? If there exists one, what is the risk-free rate r associated to this numeraire ?
3) Is a security of price $X = (4, 5, 7)$ at time $t = 1$ attainable ? Do the computation explicitly.
4) Is the market complete ? Why ?
5) Prove that the linear pricing hypothesis is satisfied.

Further assume that the linear pricing hypothesis is satisfied.

6) What is the value at time $t = 0$ of a security associated to portfolio $\theta = (2, 1, 3)$ at time $t = 1$?
7) Is there a vector π of state prices ? Justify.
8) Is there a risk-neutral probability distribution $\mathcal{Q}$?
9) Is there an arbitrage portfolio $\theta \in \mathbb{R}^3$?

Problem 2 : We consider a binomial model over two periods ($T = 2$) with $u = 1.06$, $d = 1/u$ and a risk-free rate $r = 0.02$.
We assume that the initial value of the numeraire is 1 and that the initial value of the risked security is 100.

1) Sketch the tree that corresponds to this model over two periods.
2) Show that there exists a risk-neutral probability distribution in the model over one period and give it.
3) Give the definition of a call.
4) Give the definition of a european call.
5) Give the value of a european call of strike $K = 105$ and expiration date $T = 2$.