Advanced Algorithms for Finance
Master of Financial Engineering - M2
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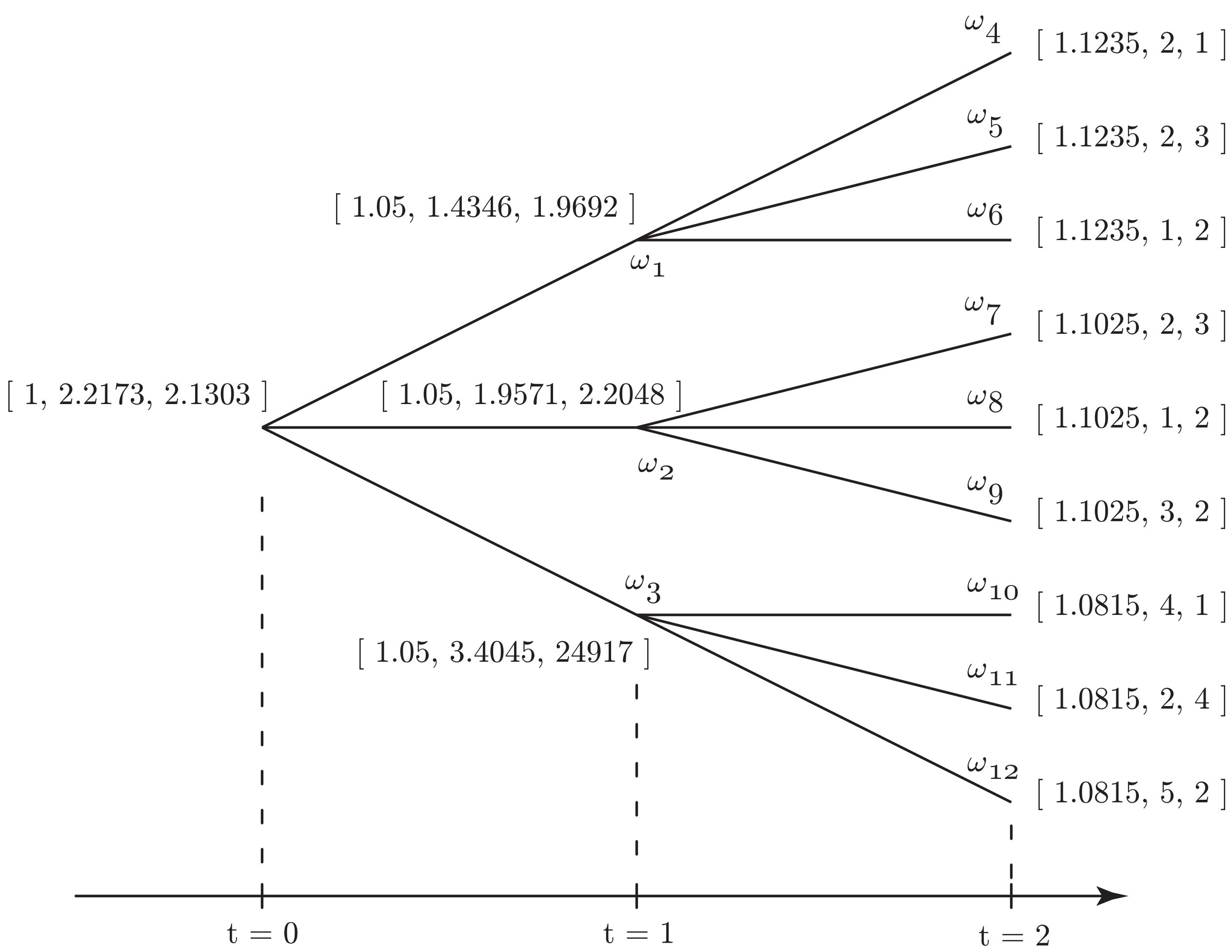
First Practical : Discrete Models

Consider a market made up of 3 securities, over two consecutive periods. This market is assumed to evolve over two consecutive periods of time. Hence, there are three instants $t = 0$, $t = 1$, and $t = 2$. The value of security i at time t in state ω_j is denoted by $S_t^{(i)}(\omega_j)$. At time $t = 1$, there are three possible states ω_1, ω_2, and ω_3. At time $t = 2$, there are nine possible states ω_4, ..., ω_{12}. This model can be represented as the following tree :



For instance, the values of the three securities at time $t = 2$ in state ω_5 are 1.1235, 2, and 3. Security $i = 0$ is the numeraire whose rate is denoted by r.

1) This model over two periods is made up of four models over one period. Identify these four models.
2) For each of these four models over one period, compute the risk-free rate r.
3) Do these four models represent complete markets ? Why ?
4) Is the linear pricing hypothesis satisfied ? Why ?
5) Compute a vector of state prices for each of the four models on one period.
6) Are there arbitrage possibilities in any of these models over one period ? Why ?
7) Compute a risk-neutral distribution for each of these models.

In the following questions, it is assumed that state ω_9 is a successor of ω_3 instead of a successor of ω_2. It is further assumed that security $i = 0$ in state ω_6 has value $S_2^{(0)}(\omega_6) = 1.0815$. Everything else remains unchanged.

8) Sketch the tree that corresponds to this new model over two periods.
9) Do these sub-models represent complete markets ? Why ?
10) Assume that the linear pricing hypothesis holds.
    For any sub-model over one period, is there any arbitrage possibility ?
    If there is an arbitrage possibility in any of these sub-models, find an arbitrage portfolio.