DICO – Dimension Coupling

3D ←→ 1D and phase transition (liquid – vapor)

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Motivation: Diesel-Injector for Passenger Cars

→ Flow characteristics during the injection of diesel:

- u = 2000 km/h
- p = 2000 bar
- t = 1 ms (later: dt = 0.1 ns → factor = 1e-10 s)

Dimension coupling for barotrop 1-phase flow

3D- Model:

$$\begin{aligned}
\partial_t \rho + \partial_x(\rho u) + \partial_y(\rho v) + \partial_z(\rho w) &= 0,\\
\partial_t(\rho u) + \partial_x(\rho u^2 + p(\rho, T_0)) + \partial_y(\rho u v) + \partial_z(\rho u w) &= 0,\\
\partial_t(\rho v) + \partial_x(\rho u v) + \partial_y(\rho v^2 + p(\rho, T_0)) + \partial_z(\rho v w) &= 0,\\
\partial_t(\rho w) + \partial_x(\rho u w) + \partial_y(\rho v w) + \partial_z(\rho w^2 + p(\rho, T_0)) &= 0,
\end{aligned}$$

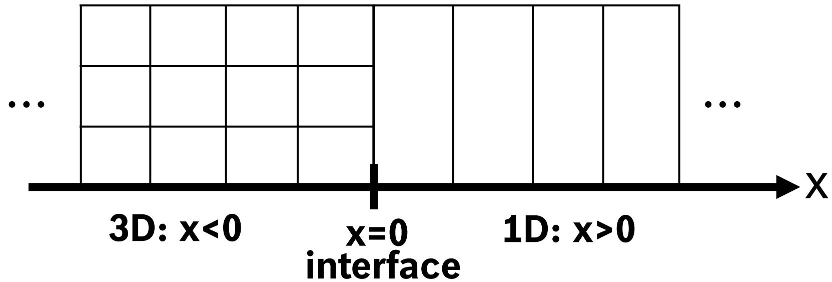
$p = p(\rho, T_0)$: barotrop (isentrop) pressure law.

1D- Model:

$$\begin{aligned}
\partial_t \rho + \partial_x(\rho u) &= 0,\\
\partial_t(\rho u) + \partial_x(\rho u^2 + p(\rho, T_0)) &= 0,
\end{aligned}$$

$p = p(\rho, T_0)$: barotrop (isentrop) pressure law.

Location of 3D-/ 1D-domain:



Dimension coupling for barotrop 1-phase flow

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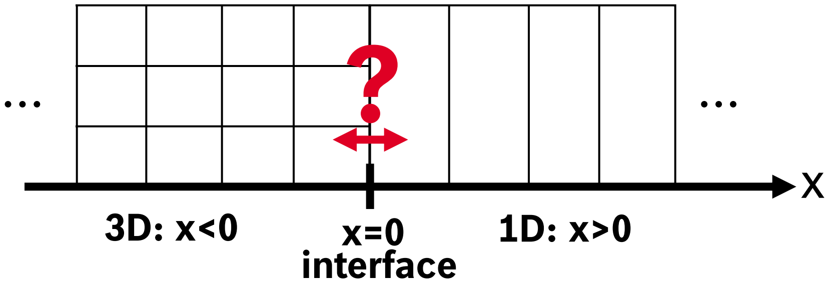
Location of 3D-/ 1D-domain:

?: Interface model

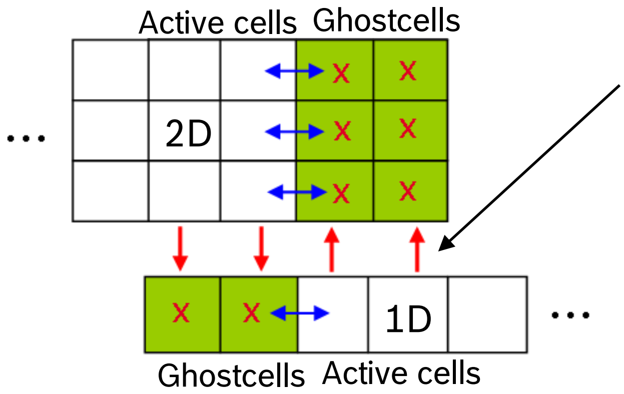


Interface-model takes into account:

3D → 1D: $(\rho_{3D}, u_{3D}, v_{3D}, w_{3D}) \leftrightarrow (\rho_{1D}, u_{1D}, 0, 0)$

PVar-Coupling: Exchange average density and velocity

Coupling strategy for PVar:



Use averaged density and velocity to calculate the fluxes in each solver at the interface.

PVar-Coupling: Exchange average density and velocity

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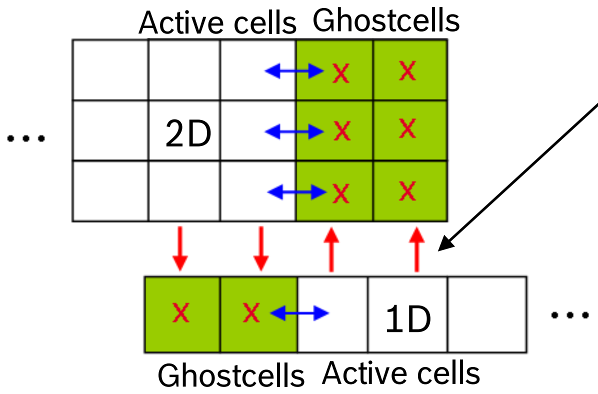
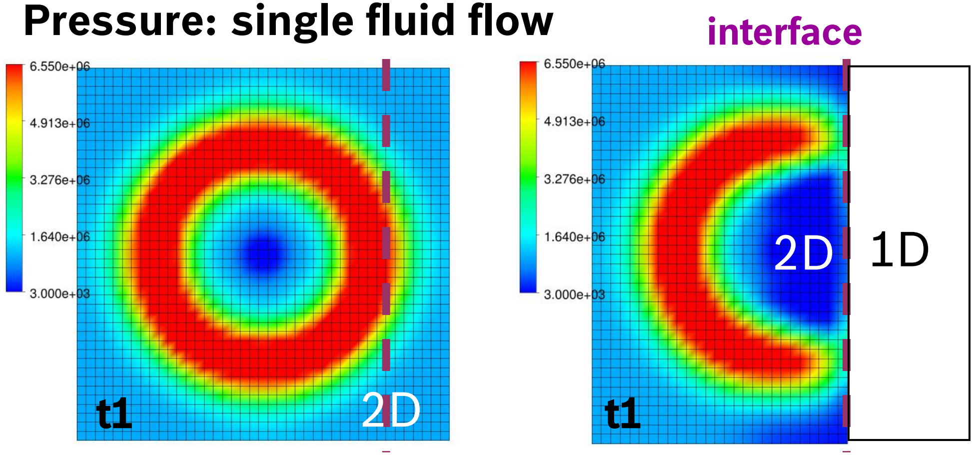


Use averaged density and velocity to calculate the fluxes in each solver at the interface.

Pressure: single fluid flow



2D-Simulation

2D1D-Cosimulation: **PVar-Coupling**

PVar-Coupling: Exchange average density and velocity

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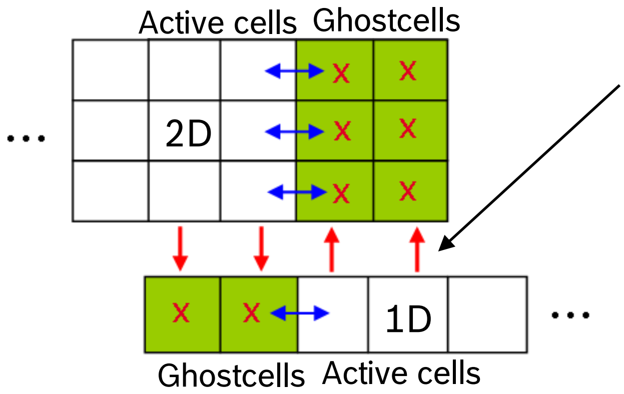
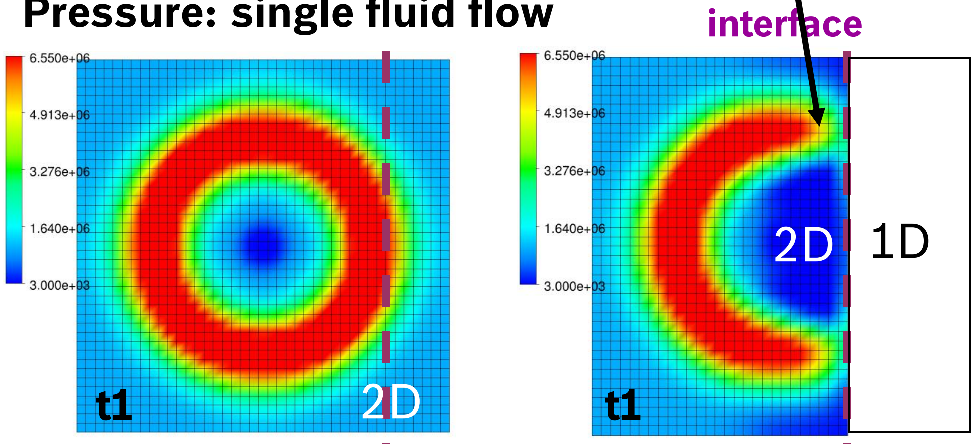


Use averaged density and velocity to calculate the fluxes in each solver at the interface.

Problem: shape is not conserved.

Pressure: single fluid flow



2D-Simulation

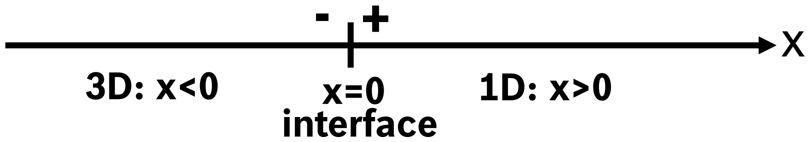
2D1D-Cosimulation: **PVar-Coupling**

Conclusion: PVar-coupling is not sufficient.

Model A (single fluid, low cross velocities)

Admissible coupling condition (necessary condition):

ρv and ρw component of numerical flux (3D-Solver) at interface $x = 0^+$ equals to zero.



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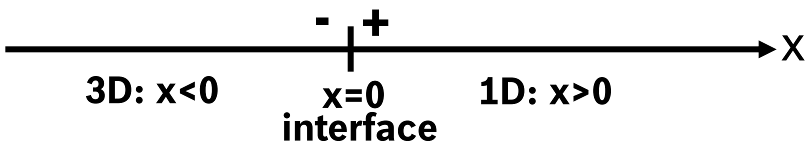

→ We need an **interface model**, which fulfills the necessary condition:

$$
\begin{aligned}
\partial_t \phi &= 0, \\
\partial_t \rho + \partial_x(\rho u) &= 0, \\
\partial_t(\rho u) + \partial_x(\rho u^2 + p(\rho, T_0)) &= 0, \\
\partial_t(\rho v) + \partial_x(\phi \rho u v) &= 0, \\
\partial_t(\rho w) + \partial_x(\phi \rho u w) &= 0.
\end{aligned}
$$

with

$$
\phi(x, y, z, t) = \begin{cases} 1, & if\ x < 0 \\ 0, & if\ x > 0. \end{cases}
$$

Model A (single fluid, low cross velocities)

Interface-model:

$$\begin{aligned}
\partial_t \phi &= 0,\\
\partial_t \rho + \partial_x(\rho u) &= 0,\\
\partial_t(\rho u) + \partial_x(\rho u^2 + p(\rho, T_0)) &= 0,\\
\partial_t(\rho v) + \partial_x(\phi \rho u v) &= 0,\\
\partial_t(\rho w) + \partial_x(\phi \rho u w) &= 0.
\end{aligned}$$

A: Single fluid flow, low cross-velocities
Thin interface with additional equation for ϕ, Roe-solver and ρ_{liq}-assumption in 1D.

Write interface model as: $\partial_t W + \partial_x F(W) = 0 \quad \longleftrightarrow \quad \partial_t W + J(W)\partial_x W = 0$

with $W = (\phi, \rho, \rho u, \rho v, \rho w)^T$ and $F(W) = (0, \rho u, \rho u^2 + p, \phi \rho u v, \phi \rho u w)^T$

Then the jacobi matrix:

$$J(W) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -u^2 + c^2 & 2u & 0 & 0 \\ \rho u v & -\phi u v & \phi v & \phi u & 0 \\ \rho u w & -\phi u w & \phi w & 0 & \phi u \end{bmatrix}$$

with eigenvalues:

$$\begin{aligned}
\lambda_1 &= 0,\\
\lambda_2 &= u - c,\\
\lambda_3 = \lambda_4 &= \phi u,\\
\lambda_5 &= u + c,
\end{aligned}$$

Model A (single fluid, low cross velocities)

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Interface-model A: $\partial_t W + J(W)\partial_x W = 0$

jacobi matrix:

$$J(W) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -u^2 + c^2 & 2u & 0 & 0 \\ \rho uv & -\phi uv & \phi v & \phi u & 0 \\ \rho uw & -\phi uw & \phi w & 0 & \phi u \end{bmatrix}$$

with eigenvectors:

$$r_1 = (\phi, 0, 0, -\rho v, -\rho w)^T$$

$$r_2 = (0, (\phi - 1)u + c, (u - c)((\phi - 1)u + c), \phi vc, \phi wc)^T$$

$$\begin{cases} r_3 = (0, 0, 0, 1, 0)^T \\ r_4 = (0, 0, 0, 0, 1)^T \end{cases}$$

$$r_5 = (0, (1 - \phi)u + c, (u + c)((1 - \phi)u + c), \phi vc, \phi wc)^T$$

Model A (single fluid, low cross velocities)

At interface: linearized Riemann-Problem → choose **Roe-Linearization.**

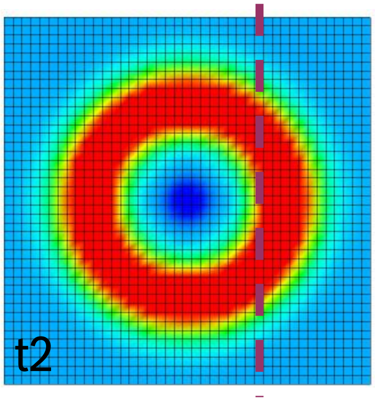
For interface-model A: $\partial_t W + J(\widehat{W})\partial_x W = 0$ where $W = (\phi, \rho, \rho u, \rho v, \rho w)^T$

and $J(\widehat{W})$ satisfies: $F(W_R) - F(W_L) = J(\widehat{W})(W_L - W_R)$.

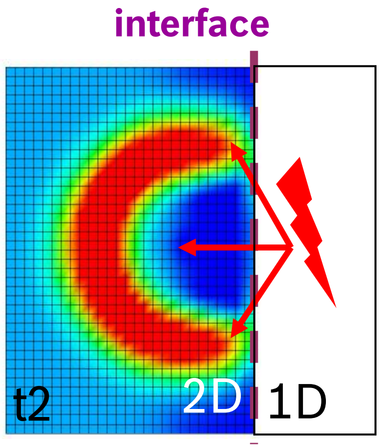
Thus we have:

$$
\begin{aligned}
\widehat{\rho} &= \sqrt{\rho_R \rho_L} \\
\widehat{u} &= \frac{\sqrt{\rho_R} u_R + \sqrt{\rho_L} u_L}{\sqrt{\rho_R} + \sqrt{\rho_L}} \\
\widehat{v} &= \frac{\sqrt{\rho_R} v_R + \sqrt{\rho_L} v_L}{\sqrt{\rho_R} + \sqrt{\rho_L}} \\
\widehat{w} &= \frac{\sqrt{\rho_R} w_R + \sqrt{\rho_L} w_L}{\sqrt{\rho_R} + \sqrt{\rho_L}} \\
\widehat{c} &= \begin{cases} \frac{c_L + c_R}{2} = c_L = c_R & , if \ \triangle\rho = 0 \\ \sqrt{\frac{\triangle p}{\triangle \rho}} & , if \ \triangle\rho \neq 0 \end{cases} \\
\widehat{\phi} &= \begin{cases} -\frac{1}{2} & , if \ u_L = 0 \\ 1 - \frac{\sqrt{\rho_R}(\sqrt{\rho_R} u_R + \sqrt{\rho_L} u_L)}{u_L(\sqrt{\rho_R} + \sqrt{\rho_L})^2} & , if \ u_L \neq 0 \end{cases}
\end{aligned}
$$

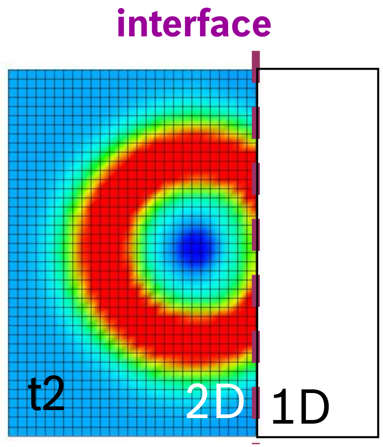
Model A/ PVar-Coupling: Testcase Pressure-Peak



3D-Simulation



3D-1D-CoSimulation
PVar-Coupling



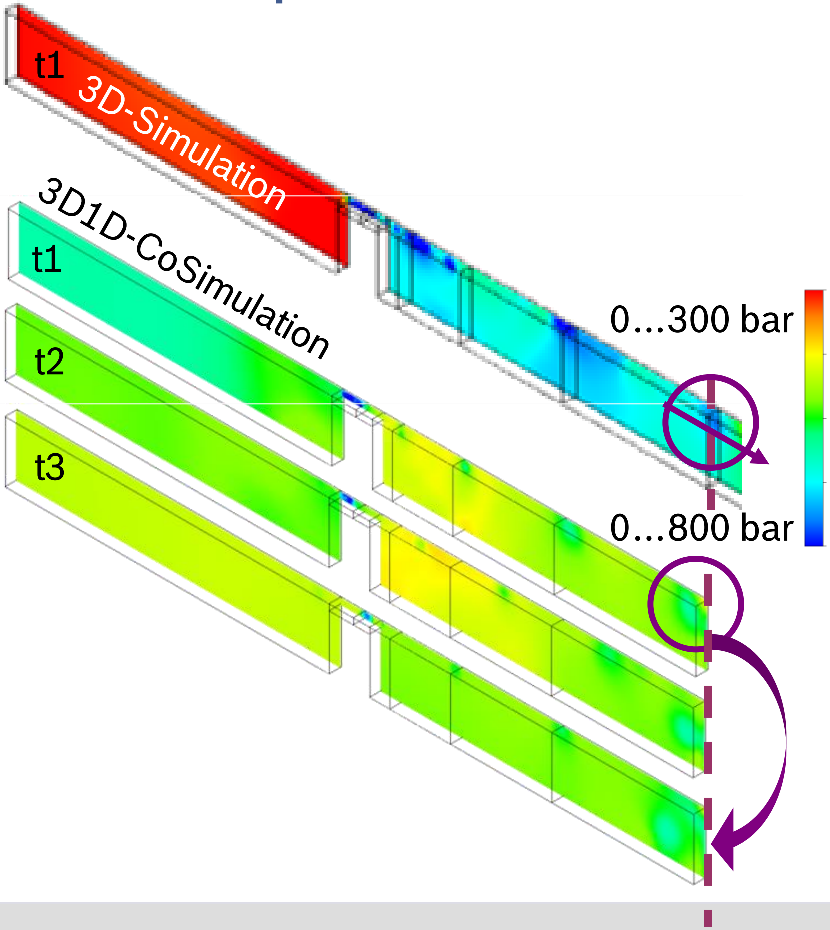
3D-1D-CoSimulation
Model A:
Flux-Coupling (Roe)

1D part

→ In the Flux-Coupling (right), the original shape of the wave is perfectly conserved.
→ **Coupling-algorithmn**: Model A with Roe-Solver for the calculation of the interface fluxes.
3D-Catum and 1D-Catum stay unchanged.

PVar-Coupling: Exchange average density and velocity

Testcase for 2-phase flow: bubbles travel over interface.



PVar-Coupling: Exchange average density and velocity

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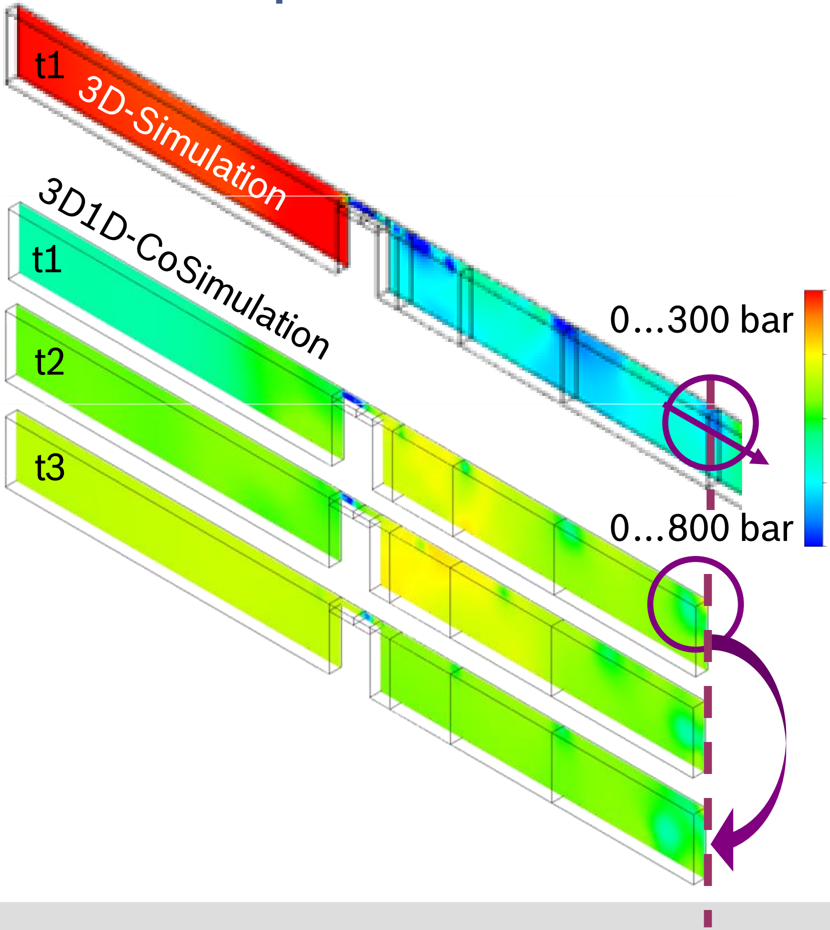


Interface-model:

3D → 1D:

$$(\rho_{3D}, u_{3D}, v_{3D}, w_{3D})$$

$$\updownarrow$$

$$(\rho_{1D}, u_{1D}, 0, 0)$$

AND

2-phase → 1-phase flow:

$$\rho = z_{liq}\rho_{liq} + z_{vap}\rho_{vap}$$

$$\updownarrow$$

$$\rho = \rho_{liq}$$

$z_i = \frac{V_i}{V_{tot}}$ the volume fraction

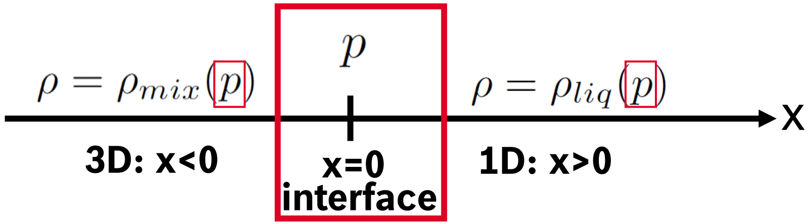
Model B1 (2-phase flow, low cross velocities)

Interface-model B1:

$$\begin{aligned}
\partial_t \phi &= 0, \\
\boxed{\partial_t p + u\partial_x p + \rho c^2 \partial_x u = 0,} \\
\partial_t u + \frac{1}{\rho}\partial_x p + u\partial_x u &= 0, \\
\partial_t v + \phi u \partial_x v &= 0, \\
\partial_t w + \phi u \partial_x w &= 0.
\end{aligned}$$

Interface: non-conservative equation to calculate **pressure**.

→ **Advantage:** constant pressure over interface.



Model B2 (2-phase flow, low cross velocities)

Interface-model B2:

$$\begin{aligned}
\boxed{\partial_t(\rho_{vap}z_{vap}) + \partial_x(\rho_{vap}z_{vap}u) + \partial_y(\rho_{vap}z_{vap}v) + \partial_z(\rho_{vap}z_{vap}w) = \lambda(g_{liq}(\rho_{liq}) - g_{vap}(\rho_{vap})),} \\
\boxed{\partial_t(\rho_{liq}z_{liq}) + \partial_x(\rho_{liq}z_{liq}u) + \partial_y(\rho_{liq}z_{liq}v) + \partial_z(\rho_{liq}z_{liq}w) = -\lambda(g_{liq}(\rho_{liq}) - g_{vap}(\rho_{vap})),} \\
\partial_t(\rho u) + \partial_x(\rho u^2 + p(\rho, T_0)) + \partial_y(\rho uv) + \partial_z(\rho uw) &= 0, \\
\partial_t(\rho v) + \partial_x(\rho uv) + \partial_y(\rho v^2 + p(\rho, T_0)) + \partial_z(\rho vw) &= 0, \\
\partial_t(\rho w) + \partial_x(\rho uw) + \partial_y(\rho vw) + \partial_z(\rho w^2 + p(\rho, T_0)) &= 0, \\
\boxed{\partial_t(z_{vap}) + u\partial_x(z_{vap}) + v\partial_y(z_{vap}) + w\partial_z(z_{vap}) = k(p_{vap}(\rho_{vap}) - p_{liq}(\rho_{liq})),}
\end{aligned}$$

where $p = z_{vap}p_{vap} + z_{liq}p_{liq}$.

3D: homogenous mixture: $\rho = z_{liq}\rho_{liq} + z_{vap}\rho_{vap}$

interface: split density in ρ_{liq} and ρ_{vap} $\longrightarrow z_i = \frac{V_i}{V_{tot}}$ the volume fraction

1D: use 2 mass conservation eqs.: ρ_{liq} and ρ_{vap}

Conclusion: Interface-models for

➔ **A: Single fluid flow, low cross-velocities**

- Thin interface with additional equation for ϕ, Roe-solver and ρ_{liq}-assumption in 1D.

→ **Single fluid flow is perfectly approximated with model A.**

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Conclusion: Interface-models for

→ A: Single fluid flow, low cross-velocities

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→ B: 2-phase flow, low cross-velocities

- **B1:** Thin interface with additional equation for ϕ,
  centered scheme with numerical viscosity and
  non-conservative pressure equation at the interface.
- **B2:** Thin interface with additional equation for ϕ, Roe-solver and
  ρ_{liq}, ρ_{vap} - mass conservation equations in 1D.

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Conclusion: Interface-models for

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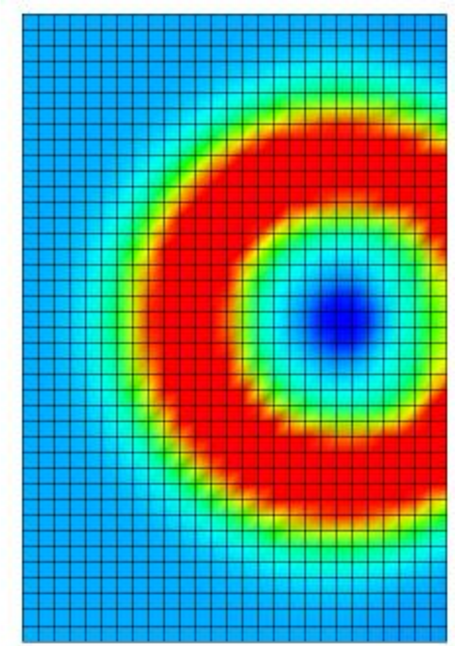
- **B1:** Thin interface with additional equation for ϕ, centered scheme with numerical viscosity and non-conservative pressure equation at the interface.
- **B2:** Thin interface with additional equation for ϕ, Roe-solver and ρ_{liq}, ρ_{vap} - mass conservation equations in 1D.

→ **C: 1-/2-phase flow, high cross-velocities**

- Model B1 or B2 with grid derefinement after the interface by factor 2 in each cell layer.

→ **2-phase and turbulent flow: it has to be shown that model B and C solve.**

Thank you for your attention!



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